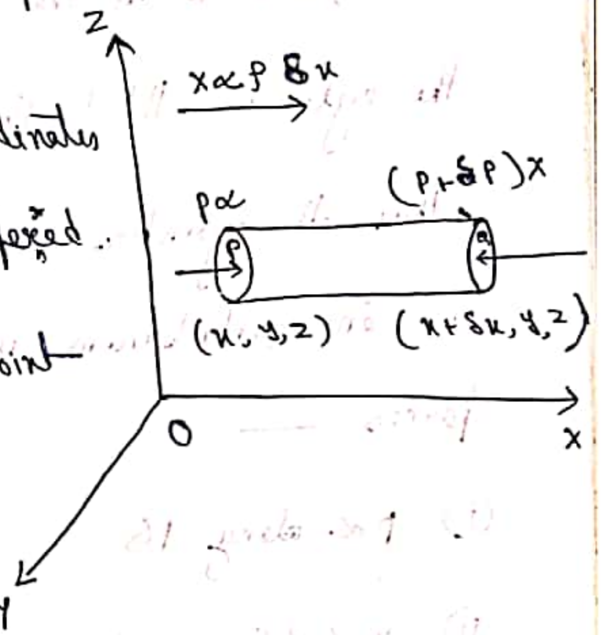
6.1. Hydro-statics.

Fluid pressure :—

Imp. Equation of pressure :—

A mass of fluid is at rest under the action of given forces. Find the equation whose determins the pressure at any point of the fluid.

Soln



Let (x, y, z) be the co-ordinates of a point P in the fluid refered to rectangular axies. We take a point Q very near P s.t. PQ is parallal to the axies of x. Let the co-ordinate of Q be $(x+\delta x, y, z)$

Describe a small cylinder about PQ as axies having ~~end~~ its plane ends perpendicular to PQ. Let α be the area of either plane end of the cylinder also let p be the pressure at P and $p+\delta p$ the pressure at Q

$\therefore$ Thrust at the ~~ten~~ plane end at P is $p\alpha$

$$\left[\text{pressure} = \frac{\text{Force}}{\text{area}} = \frac{\text{Thrust}}{\text{area}}\right.$$

$$\Rightarrow \text{Thrust} = \underline{\underline{\text{pressure} \times \alpha}} = p\alpha$$

and that at Q is $(P+\delta P)\alpha$

Now, ρ be the mean density of the cylinder PQ

then mass of the cylinder $= \rho \alpha \delta x$

If X, Y, Z be the given component forces per unit mass then $X \alpha \rho \delta x$ is the force ~~of~~ on the cylinder $\parallel^{el}$ to the axes of X.

Thus the cylinder about PQ axes is in equilibrium under the forces —

Density $= \dfrac{\text{Mass}}{\text{volumn}}$

$= \dfrac{\text{Mass}}{\text{area}}$

$\rho = \dfrac{\text{mass}}{\alpha \delta x}$

$\Rightarrow \text{Mass} = \rho . \alpha . \delta x$

(i) $P\alpha$ along PQ

ii) $x \alpha \rho \delta x$ along PQ and

iii) $(P+\delta P)\alpha$ along QP as shown in the fig.

and for the equilibrium of cylinder we have

$$(P+\delta P)\alpha = P\alpha + X \rho \alpha \delta x$$

$$\Rightarrow \delta P \alpha = X \rho \alpha \delta x$$

$$\Rightarrow \frac{\delta P}{\delta x} = X \rho$$

proceeding to limits as $\delta x \to 0$, we get

$$\frac{\partial P}{\partial x} = \rho X \longrightarrow (1)$$

IIly we have $$\left.\begin{aligned} \frac{\partial p}{\partial y} &= \rho Y \\ \frac{\partial p}{\partial z} &= \rho Z \end{aligned}\right\} \longrightarrow \text{(ii)}$$

But we always have

$$\cancel{dp} \quad dp = \frac{\partial p}{\partial x}\, dx + \frac{\partial p}{\partial y} \cdot dy + \frac{\partial p}{\partial z}\, dz$$

$$= \rho X dx + \rho Y dy + \rho Z dz \quad \cancel{\longrightarrow (iii)}$$

$$= \rho\,(X dx + Y dy + Z dz) \longrightarrow \text{(iii)}$$

This is the diffⁿ eqⁿ determining pressure p at a point.