The Harrod-Domar Growth Model

The Harrod-Domar models of economic growth are based on the experiences of advanced capitalist economies to analyse the requirements of steady growth in such economy. The Harrod-Domar economic growth model stresses the importance of savings and investment as key determinants of growth. The model emphases on the dual character of investment:

1. It creates income which is regarded as the 'demand effect'.
2. It augments the productive capacity of the economy by increasing its capital stock which is regarded as the 'supply effect' of investment.

The main assumptions of the Harrod-Domar models are as follows:

1. A full-employment level of income already exists.
2. There is no government interference.
3. The model is based on the assumption of closed economy.
4. There are no lags in adjustment of variables.
5. The average propensity to save (APS) and marginal propensity to save (MPS) are equal to each other. Symbollically, $S/Y = \Delta S/\Delta Y$
6. Both propensity to save and "capital coefficient" (i.e., capital-output ratio) are given constant.
7. Income, investment, savings are all defined in the net sense and hence they are considered over and above the depreciation.
8. Saving and investment are equal in ex-ante as well as in ex-post sense.

Given the above main general assumptions, we shall discuss both models separately as below. Although Harrod and Domar models differ in some aspects, they are similar in substance as both the models stress the essential conditions of achieving and maintaining steady growth.

The Harrod Model:

An English economist, Henry Roy Forbes Harrod (13 February 1900 – 8 March 1978) tries to show in his model how steady growth may occur in the economy. Once the steady growth rate is interrupted and the economy falls into disequilibrium, cumulative forces tend to perpetuate this divergence thereby leading to either secular deflation or secular inflation.

The Harrod Model is based upon three distinct rates of growth as below:

1. The actual growth rate (G)
2. The warranted growth rate (G_w)
3. The natural growth rate (G_n)

1. The actual growth rate (G): It is defined as the ratio of change in income (ΔY) to the total income (Y) in the given period. Mathemaically; $G = \Delta Y/Y$

The actual growth rate (G) is determined by:

(a) Saving-Income ratio (s) known as the Average Propensity to Save which is expressed as $s = S/Y$

(b) Capital- Output ratio (C) which is expressed as $C=\Delta K/\Delta Y$ where ΔK denotes change in Capital stock which equal investment (I)

The relationship between the actual growth rate and its determinants is expressed as: GC = s ------(1)

Now;

Since

$$G = \frac{\Delta Y}{Y}$$

$$C = \frac{\Delta K}{\Delta Y} = \frac{I}{\Delta Y} \qquad [\because \Delta K = I]$$

Because $s = \frac{S}{Y}$

Substituting the value of G, C, and s in equation (1), we get

$$\frac{\Delta Y}{Y} \times \frac{I}{\Delta Y} = \frac{S}{Y}$$

or

$$\frac{I}{Y} = \frac{S}{Y}$$

or $I = S$

The above equation so derived explains that the condition for achieving the steady state growth is that ex-post (actual, realized) savings must be equal to ex-post investment.

2. The warranted growth rate (G_w): Warranted growth Rate also known as Full-capacity growth rate refers to that growth rate of the economy when it is working at full capacity. In other words, G_w is interpreted as the rate of income growth required for full utilization of a growing stock of capital.

Warranted growth rate (G_w) is determined by capital-output ratio and saving- income ratio and their relationships is expressed as:

$$G_w C_r = s$$

or $Gw=s/Cr$

where ;

C_r denotes the amount of capital-output ratio needed to maintain the warranted

s denotes the saving-income ratio.

The above equation reflects that if the economy is to advance at the steady rate of Gw at its full capacity, income must grow at the rate of s/Cr per year.

3. The natural growth rate (G_n): The natural growth rate also known as the potential or the full employment rate of growth is the rate of economic growth required to maintain full employment. The natural growth rate regarded as 'the welfare optimum' by Harrod is the maximum growth rate which an economy can achieve with its available natural resources.

The Natural growth rate is determined by natural conditions such as labor force, natural resources, capital equipment, technical knowledge etc. The third fundamental relation in Harrod's model showing the determinants of natural growth rate is expressed as: G_nC_r = or $\neq s$

Condition for the Achievement of Steady Growth:

According to Harrod, the economy can achieve steady growth when there is equality between G and G_w at the same time between C and C_r. This condition can be expressed as:

$G = G_w$ and $C = C_r$

Harrod states that a slight deviation of G from G_w will lead the economy away and further away from the steady-state growth path. Thus, the equilibrium between G and Gw at this junction is considered as a knife-edge equilibrium.

Instability of Growth:

As discussed above, to achieve steady growth in economy, a balance between G and G_w must be maintained otherwise the economy will be in disequilibrium. Therefore, Harrod analysed two situations when equilibrium condition is not satisfied:

(*i*) If $G > G_w^*$ then $C < C_r$ (*ii*) If $G < G_w$ then $C > C_r$

The first situation implies that if such situation occurred, the economy will find itself in the quagmire of inflation. This is because under this situation, the growth rate of income being greater than the growth rate of output, the demand for output would exceed the supply of output.

In contrast, the second situation implies if such situation occurred, the economy will lead to secular stagnation because actual income grows more slowly than what is required by the productive capacity of the economy leading to an excess of capital goods (C>Cr).

For once if steady growth equilibrium path is disturbed, it is not self-correcting. Therefore, it follows that one of the major tasks of public policy is to bring G and Gw together in order to maintain long-run stability. For this purpose, Harrod introduces his third concept of the natural rate of growth. The whole argument can also be shown with the help of the following diagram:

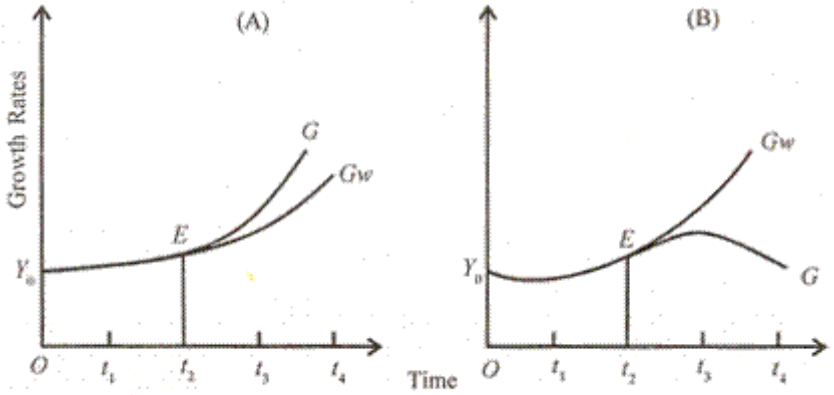


Fig. 1.

As shown in Panel –(A) of the above figures, starting from the initial full employment level of income Y_0, the actual growth rate G follows the warranted growth path Gw up to point E through period t_2. However, from t_2 onward G deviates from Gw and is higher than the latter. In subsequent periods, the deviation between the two becomes larger and larger.

As shown in Panel–(B), from period t_2 onward, G deviates from Gw where G falls below Gw and the two continue to deviate further away in subsequent periods.

Interaction of G, G_w and G_n:

To achieve full employment equilibrium growth, the economy must satisfy the condition where Gn=Gw = G. But this is a knife-edge balance. For once there is any divergence between natural, warranted and actual rates of growth conditions of secular stagnation or inflation would be generated in the economy. The same argument can be shown through the following diagram:

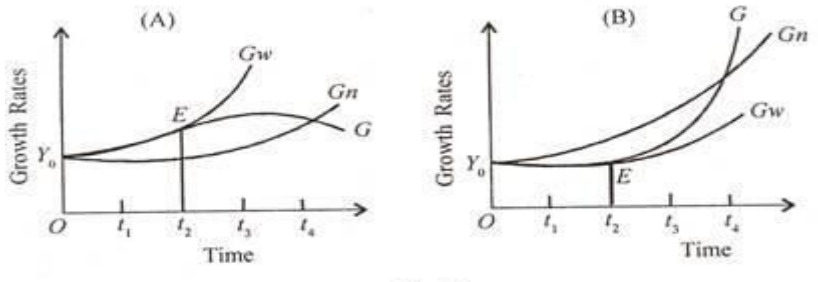


Fig. 2.

As shown in Panel-(A), if Gw>Gn, secular stagnation will develop resulting in unemployment. In such a situation, Gw is also greater than G for most of the time because the upper limit to the actual rate is set by the natural rate.

If Gw < Gn, secular inflation will develop in the economy. In such a situation, Gw is also less than G for most of the time as the one shown in Panel-(B) of the above diagram.

The instability in Harrod's model is due to the rigidity of its basic assumptions such a fixed production function, a fixed saving ratio, and a fixed growth rate of labor force. The policy implications of the model are that saving is a virtue in any inflationary gap economy and vice in a deflationary gap economy. Thus, in an advanced economy, s has to be moved up or down as the situation demands.

The Domar Model:

A Russian American economist, Evsey David Domar (April 16, 1914 – April 1, 1997), builds his model from both demand as well as the supply side based on dual effect of investment and provided the solution for steady growth.

To simplify the model, the demand and the supply equation in the incremental form can be written as follows:

The demand side of the long-term effect of investment can be summarized and expressed through the following relation as:

$\Delta Y_d = \Delta I\ (1/\alpha)$ [Change in income (ΔY_d) equals multiplier ($1/\alpha$) times the Change in investment (ΔI)]

Or $\Delta Yd = \frac{\Delta I}{\alpha}$ ……(1)

Where;

α (Alpha) = Marginal propensity to save which is reciprocal of multiplier.

The supply size of investment can be summarized and expressed through the following relation as:

$\Delta Y_s = \sigma \Delta K$ [Change in output supply (ΔY_s) equals the product of Change in real capital (ΔK) and capital Productivity (σ)]

Or $\Delta Y_s = \sigma I$……(2) [Since $\Delta K=I$ where I denotes Net investment]

Equilibrium for Steady Growth:

For achieving steady growth, aggregate demand and aggregate supply must be balanced as expressed below:

$\Delta Y_d = \Delta Y_s$ ……(3)

By substituting the value of ΔY_d and ΔY_s from equations (1) and (2) respectively in equation (3), we get:

$$\frac{\Delta I}{\alpha} = \sigma I$$

Or $\Delta I = \alpha\sigma I$……(4) [By Cross multiplying]

Dividing both sides of equation (4) by I, we get;

$$\frac{\Delta I}{I} = \alpha\sigma$$

Or $\frac{\Delta Y}{Y} = \alpha\sigma$……(5) $\left[Since\ by\ assumption\ of\ the\ model\ \frac{\Delta I}{I} = \frac{\Delta Y}{Y}\right]$

The last Equation(5) explains that if steady growth is to be maintained, the income growth rate ($\Delta Y/Y$) should be equal to the product of marginal propensity to save (α) and the productivity of capital (σ)(sigma).

Domar's condition of steady state growth can be explained with the help of numerical example. Suppose the productivity of capital (σ) is 25% and the marginal propensity to save ((α) is12%, then;

$$\frac{\Delta Y}{Y} = \frac{25}{100} \times \frac{12}{100} = \frac{3}{100}\ or\ 3\%$$

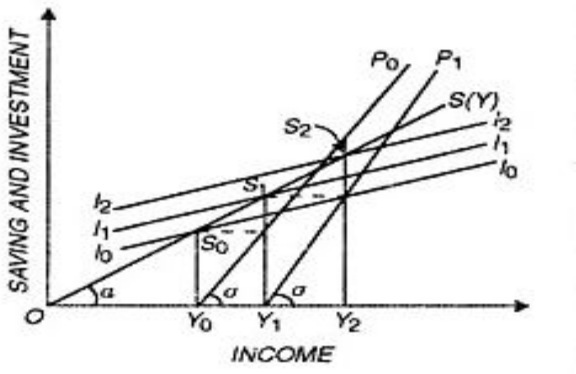
Or $\frac{\Delta I}{I} = \frac{25}{100} \times \frac{12}{100} = \frac{3}{100}\ or\ 3\%$

Thus, the above numerical example shows that income and investment must grow at an annual rate of 3% if steady growth rate is to be maintained at full-employment. Any divergence from this 'golden path' will lead to cyclical fluctuations. Disequilibrium reflecting non-steady growth state would prevail if:

1) $\frac{\Delta I}{I} > \alpha\sigma$ and the economy would experience inflation.
2) $\frac{\Delta I}{I} < \alpha\sigma$ and the economy would suffer from secular stagnation.

Diagram Representation of Domar Model:

Domar Model can also be explained with the help of the following diagram as below:



As shown in the above figure, the line S(Y) passing through the origin indicates the level of saving corresponding to different levels of income. I_oI_o, I_1I_1 and I_2I_2 are the various levels of

investment. Y_oP_o and Y_1P_1 measure the productivity of capital corresponding to different levels of investment. The lines Y_oP_o and Y_1P_1 are drawn parallel so as to show that productivity of capital remains unchanged. The level of income Y_o is determined by the intersection of saving line S(Y) and the investment line I_oI_o. At the level of income Y_o, the saving is Y_oS_o. When the saving Y_oS_o is invested, it will increase the income level from OY_o to OY_1. The productive capacity will also rise correspondingly. The extent of the income increase depends upon the productivity of capital, which is measured by the slope of the line Y_oP_o (α). Higher is the level of income higher the productive capacity. Similarly, when the level of income is OY_1 the level of saving is S_1Y_1. With investment of S_1Y_1 income will further rise to the level Y_2. This increase in income means expansion of purchasing power of the economy.

COMPARISON OF HARROD MODEL AND DOMAR MODEL

Similarities:

(i) The models are based on similar assumptions. It is for this reason that the names of Harrod and Domar are clubbed in any discussion of growth models.

(ii) Both the models employ Keynesian saving-investment equality as a condition for steady growth.

(iii) Both these models stress the "Knife-edge equilibrium".

(iv) Both the models have been built in the context of advanced economies where capital is found in abundance.

(v) As against Keynes' macro-static theory, Harrod and Domar hold that a dynamic approach to growth should be introduced in the long run.

Dis*similarities:*

(1) Domar assigns a key role to investment in the process of growth while Harrod regards the level of income as the most important factor in the growth process.

(2) Domar forges a link between demand and supply of investment while Harrod equates demand and supply of saving.

(3) The Domar model is based on one growth rate αϬ. But Harrod uses three distinct rates of growth: the actual rate (G), the warranted rate (Gw) and the natural rate (Gn).

(4) Domar gives expression to the multiplier but Harrod uses the accelerator about which Domar appears to say nothing.

(5) Domar's assumption that $\Delta I/I = \Delta Y/Y$. But Harrod does not make such assumptions.