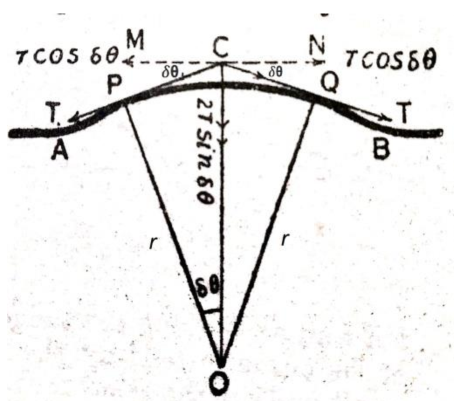
<u>**Physics-HC-2060 (wave-optics) 2nd Semester Honours.**</u>

<u>Unit-IV-Velocity of waves</u>

Velocity of Transverse vibrations of stretched strings.

A string is a cord whose length is very large as compared to its diameter and which is perfectly uniform and flexible. When a string is stretched between two points and is plucked in a directionat right angles to its length transverse vibrations are produced in it. The particles vibrate to perpendicular to its length and movement is handed on from particle to particle. A transverse wave travels along the string with a velocity depending upon certain constants of the string.

Let us consider a part of the wave *AB* travelling in the string from left to right with a velocity *v*. If the string is pulled from right to left with the same velocity the wave *AB* will remain stationary with respect to paper in space. A small part *PQ* of the wave *AB* can be considered to be the arc of a circle. As the string moves along the circular are *PQ* a centripetal force acts on it along the radius toward the centre *O*.



A uniform tension or stretching force *T* acts in the string throughout. This tension *T* acts along *CP* at *P* and along *CQ* at *Q*, where *CP* and *CQ* are tangents to the arc *CQ* at *P* and *Q* respectively. Draw *PO* and *QO* perpendiculars to the arc at *P* and *Q* meeting at *O* the centre of the circular are and let its radius be *r*.

Let the angle *POQ* be denoted by $2\delta\theta$. Join *OC*; then *OC* is the bisector of the angle *POQ*.

$\therefore$ arc $PQ = r \times 2\delta\theta$

Let *m* be the mass per unit length of the string, then

Mass of the part $PQ = m \times PQ$

$$= 2mr\delta\theta.$$

The centrifugal force acting on *PQ*

$$= \frac{2mr\delta\theta.v^2}{r}$$

$$= 2mv^2\delta\theta$$

The tension *T* can be resolved into two rectangular components. The horizontal components, $T\cos\delta\theta$ along *CM* and $T\cos\delta\theta$ along *CN,* cancel each other being equal and opposite. The vertical components of tension act in the same direction and total force along

$$CO = 2T\sin\delta\theta = 2T\,\delta\theta$$

as $\delta\theta$ is very small. This provides the necessary centripetal force.

The centripetal force acting outwards is balanced by this force which acts along *CO* towards *O* and in the equilibrium position.

$$2T\,\delta\theta = 2mv^2\,\delta\theta$$

or $$T = mv^2$$

$$\therefore \qquad v = \sqrt{\frac{T}{m}}$$

Hence the velocity of a transverse wave along a straught string varies directly as the square root of the tension *T* and inversely as the square root of the linear density, i.e., the mass per unit length *m*.