Velocity of Longitudinal waves in a Fluid in a pipe.

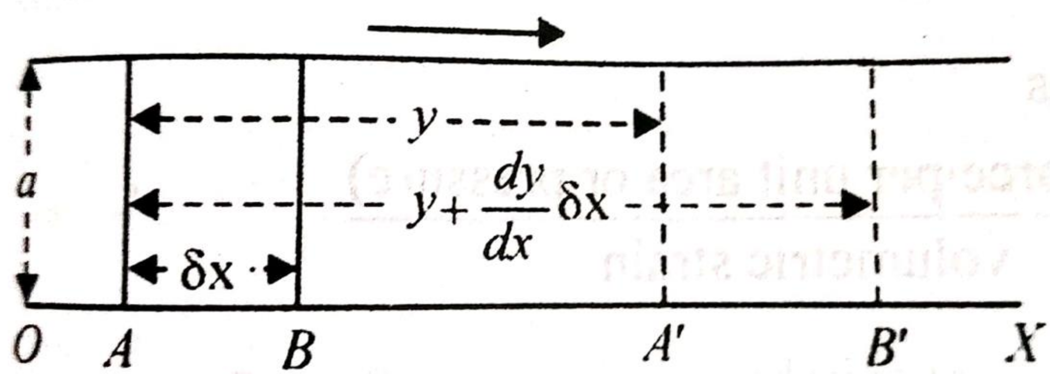
Velocity of longitudinal waves in a fluid is given by

$$v = \sqrt{\frac{E}{\rho}}$$

Where E is the coefficient of volume elasticity (bulk modulus) of the medium and ρ its density.

Let us consider longitudinal sound waves are travelling in a fluid medium from left to right. Due to these longitudinal waves rarefactions and condensations will be formed in the medium. Let these rarefactions and condensations travel from left to right with a velocity v along the X-axis OX in a pipe of area of cross- section a.

Now we consider two planes perpendicular to OX at A and B a small distance δx apart. When sound waves travel through the medium let at any instant the plane A be displaced to the position A' and plane B to the position B' .



Now the displacement of the plane at A = AA' = y, the rate of change of displacement = $\frac{dy}{dx}$

∴ Displacement at $B = BB' = y + \left(\frac{dy}{dx}\right) \delta x$

As the displacement of the plane *B* is greater than the displacement of the plane *A*, the distance *B'A'* is greater than *BA* by an amount given by

$$Y + \left(\frac{dy}{dx}\right) \delta x - y = \left(\frac{dy}{dx}\right) \delta x$$

∴ Change in volume $= a \left(\frac{dy}{dx}\right) \delta x$, original volume of the layer $AB = a\, \delta x$

∴ Volumetric strain $= \frac{a\left(\frac{dy}{dx}\right)\delta x}{a\delta x} = \frac{dy}{dx}$

Let *P* be the excess of pressure over the atmosphere at the plane *A*. Suppose the excess pressure varies with distance *x* at the rate $\frac{dP}{dx}$, then

Excess pressure at the plane $B = P + \left(\frac{dP}{dx}\right) \delta x$

The excess of pressure at *A* and *B* act in opposite directions.

∴ The resultant pressure acting on the element *AB*

$$= P + \left(\frac{dP}{dx}\right) \delta x - P = \left(\frac{dP}{dx}\right) \delta x$$

Hence forcw acting on the element $AB = a \left(\frac{dP}{dx}\right) \delta x$

∴ The mass of the element $AB = a\, \delta x\, \rho$, here ρ is the density of the medium.

If $\frac{d^2y}{dt^2}$ is the acceleration of the mass of the element AB, then (according to Newton's second law of motion)

$$a\,\delta x\,\rho\frac{d^2y}{dt^2} = a\left(\frac{dP}{dx}\right)\delta x$$

or $$\rho\frac{d^2y}{dt^2} = \frac{dP}{dx} \qquad \text{....(i)}$$

Now elasticity of a medium $E = \frac{stress}{strain} = \frac{P}{\frac{dy}{dx}}$

∴ $$P = E\frac{dy}{dx}$$

or $$\frac{dP}{dx} = \frac{d}{dx}E\left(\frac{dy}{dx}\right) = E\frac{d^2y}{dx^2} \qquad \text{....(ii)}$$

Comparing (i) and (ii), we have

$$\rho\frac{d^2y}{dt^2} = E\frac{d^2y}{dx^2}$$

or $$\frac{d^2y}{dt^2} = \frac{E}{\rho}\frac{d^2y}{dx^2}$$

This is the differential equation of a wave motion of a longitudinal wave propagating in a fluid medium. The velocity is given by $v = \sqrt{\frac{E}{\rho}}$

This shows that the velocity of sound (or longitudinal) waves through a fluid depends upon volume elasticity and density of the fluid.