Space Dynamics: unit (I, II)

Basic definition of (1) Conservative Force:

A Force $\vec{F}$ is said to be conservative if the work done by $\vec{F}$ in displacing a particle from the position A to another position B is independent of the path joining the points A and B

OR

A force $\vec{F}$ is said to be conservative if $\exists$ a continuously differentiable scalar point function V such that $\vec{F} = -\vec{\nabla} V$ Here V is called the force potential or potential energy.

(2) A force $\vec{F}$ is said to be conservative iff $\vec{\nabla} \times \vec{F} = 0$

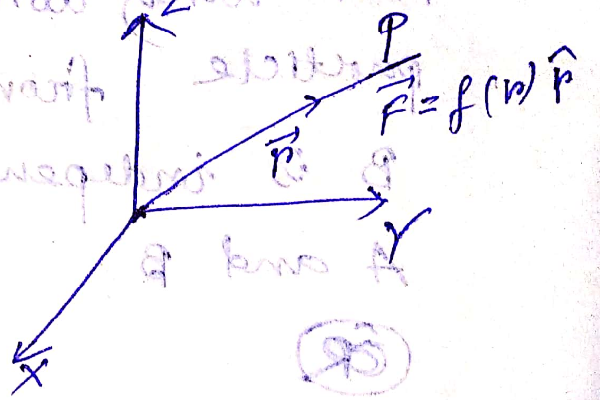
(3) A force $\vec{F}$ is said to be conservative if $\oint_c \vec{F} \cdot d\vec{r} = 0$

(4) Central force: A force acting on a particle

of mass m is said to be central force with O as the centre of force if

(i) it is always directed from m towards or away from the fixed point O

(ii) its magnitude depends only on the distance r from O.



In symbols, A force $\vec{F}$ is said to be a central force if it can be expressed as

$$\vec{F} = f(r)\hat{r} = f(r)\frac{\vec{r}}{r} = \phi(r)\vec{r}$$

(5) A central force is conservative

(6). A central orbit is a plane curve

(7). Newton's law of Gravitation: Any two particles of masses m_1 and m_2 placed at a distance r apart are attracted toward each other with force $\vec{F} = -\frac{G \cdot m_1 m_2}{r^2}$

8. Force of Gravitation is conservative

9. Gravitational potential :

Let us consider the gravitational force $\vec{F} = -\frac{G M . m}{r^2} . \hat{r}$ on the particle of mass m at $P(\vec{r})$ due to the mass M at O.

Let V be the force potential corresponding to $\vec{F}$

$$\therefore \vec{F} = -\vec{\nabla} V \qquad \because \vec{\nabla} = \frac{d}{dr}$$

$$\Rightarrow \vec{F} . d\vec{r} = -\vec{\nabla} V . d\vec{r}$$

$$= -dV$$

$$\Rightarrow \therefore dV = -\vec{F} . d\vec{r} = \frac{G . M . m}{r^2} . \hat{r} . d\vec{r}$$

$$= \frac{G . M . m}{r^3} . \vec{r} . d\vec{r}$$

$$= \frac{G . M . m}{r^3} r \, dr$$

$$= \frac{G . M . m}{r^2} dr$$

$$\therefore V = -\frac{GMm}{r}$$