Newton's Formula of a velocity of sound, Laplace's correction

The general expression for the velocity, *v* of a wave in a gaseous medium of density ρ and bulk modulus of elasticity *E*,

$$V = \sqrt{\frac{E}{\rho}}$$

Newton assumed that the propagation of sound waves in a gas take place under ***isothermal condition***, when the bulk elasticity of a gas equals its pressure. Hence, the above formula can be written as

$$V = \sqrt{\frac{P}{\rho}}$$

where *P* is the pressure of the gas.

[Considering a mass of gas at initial pressure *P* and volume *V*. Let the pressure of this mass of gas be increased by a small amount *p* and let the consequent volume be *v*. According to Boyle's law,

$$P.V = (P + p)\,(V - v)$$

$$P.V = P.V - P.v + p.V - p.v$$

$$P.v = p.V \qquad \text{(p.v being very small may be neglected)}$$

Or $$P = \frac{p.V}{v} = \frac{p}{\frac{v}{V}}$$

Here, *p* is the volume stress and $\frac{v}{V}$ is the volume string.

Hence, $P = \frac{volume\ stress}{volume\ string} = Bulk\ modulus$]

Error in Newton's formula for velocity of sound in air at N.T.P.

Normal air pressure = 76 cm Hg.= 76 x 13.59 x 980 dynes/cm^2 (*P)*.

At 0^{o}C and at normal atmospheric pressure, the density of air = 0.001293 g/cm^3 .

So, from Newton's formula, V = $\sqrt{\frac{76 \text{ x } 13.59 \text{ x } 980}{0.001293}}$ = 280 m/s

This calculated value of velocity of sound in air deffers from experimental value (332 m/s).

Laplace's correction of Newton's formula

This differences between the experimentally observed value of velocity of sound in air and the calculated value was explained by Laplace. He pointed out that during propagation of sound wave, each layer of the medium is alternately compressed and rarefied. During compression the layer is heated and during rarefaction the layer is cooled. On account of the fact, that air is a bad conductor of heat as well as a bad radiator, heat cannot flow quickly from the heated layers to the cooled layers. Consequently, propagation of sound waves takes place under a ***adiabatic consition***. Under adiabatic condition,

$P.V^{\gamma}$ = constant, where γ is the ratio of specific heat at constant pressure to the specific heat at constant volume.

Using the above relation Laplace showed that

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

For air γ = 1.41. Substituting the value of γ, we get V = 332 m/s.

This result agrees with the experimental value.

Effect of pressure: The velocity of sound is independent of pressure. Let us consider a mass m having volume V and density ρ then $V = \frac{m}{\rho}$.

According to Boyle's PV = constant

or $\frac{P}{\rho}m$ = constant

As the mass of the gas does not change due to variation in temperature and pressuere

$\therefore$ $\sqrt{\frac{P}{\rho}} =$ constant, the velocity of sound in a gas is independent of pressure.

Effect of density: The velocity of sound in a gas is inversely proportional to the square root of its density.

Effect of temperature: The velocity of sound is directly proportional to the square root of the absolute temperature.

Effect of wind: When the wind is blowing in the same direction in which the sound is travelling, the velocity of sound increases and when the wind is blowing in the opposite direction, velocity of sound decreases.

Effect of moisture: Water vapour is lighter than water. The presence of moisture in air changes its density. Since the density of moist air is less than that for dry air sound travels faster in moist air than in dry air.