Collision (Scattering)

Inelastic collision between two particles in the laboratory frame:

When the colliding particles stick together after the collision and move as a single particle, the collision is a completly inelastic collision.

In the laboratory frame, let the velocity of the particle of mass m_1 is $\vec{u}_1$ and m_2 is at rest. Let $\vec{V}$ be the velocity of the combined mass $(m+m_2)$ after collision. By the law of conservation of momentum we have,

$$m_1\vec{u}_1 + m_2 \cdot 0 = (m_1+m_2)\vec{V}$$

$$\therefore V = \frac{m_1 u_1}{m_1+m_2} \longrightarrow (1)$$

Both particles moves with same velocity after the collision.

Now kinetic energy of the particles before collision

$$K_1 = \frac{1}{2} m_1 u_1^2 \longrightarrow (2)$$

Kinetic energy after collision.

$$K_2 = \frac{1}{2}(m_1+m_2)V^2 \longrightarrow (3)$$

$$\therefore \frac{K_2}{K_1} = \frac{(m_1+m_2)V^2}{m_1 u_1^2}$$

But $v^2 = \frac{m_1^2 u_1^2}{(m_1+m_2)^2}$ from equ^n ①

$$\therefore \frac{K_2}{K_1} = \frac{m_1^2 u_1^2}{(m_1+m_2)^2} \cdot \frac{m_1+m_2}{m_1 u_1^2}$$

$$\frac{K_2}{K_1} = \frac{m_1}{m_1+m_2} \rightarrow ④$$

Since $m_1 < m_1 + m_2$ so $K_2 < K_1$. This shows that in inelastic collision, kinetic energy is lost. This lost energy may appear as heat and sound energy etc. This lost fraction is $1 - \frac{K_2}{K_1} = 1 - \frac{m_1}{m_1+m_2}$

$$= \frac{m_2}{m_1+m_2}$$

Inelastic collision between two particles in centre of mass frame:

In centre of mass frame of reference the velocity of centre of mass of the system is

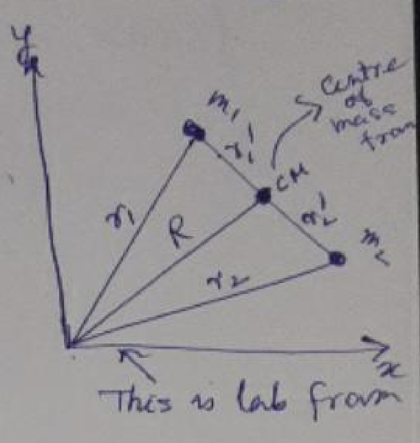
for lab frame

$$\vec{U}_{cm} = \frac{m_1 \vec{u}_1}{m_1+m_2}.$$

$\bar{U}_{cm}$ is the velocity of cm observed by an observer in lab frame

By defination the velocity of the centre of mass is zero for an observer standing at centre of mass.

For observer at lab frame the centre of mass moves with velocity $\vec{u}_{cm}$. But for the observer at centre of mass $V_{cm} = 0$ since $R'_{cm} = 0$



Whenever the particle of mass m_1 moves with velocity $\vec{u}_1$ before collision then the centre of mass will moves with velocity $\vec{u}_{cm}$. So for the observer at centre of mass the initial velocity of m_1 is

$$\vec{u}_1' = \vec{u}_1 - \vec{u}_{cm}$$

$$= u_1 - \frac{m_1 u_1}{m_1 + m_2}$$ and that of

m_2 is $\vec{U}_2' = \vec{u}_2 - \vec{u}_{cm} = -\vec{u}_{cm}$

$$= -\frac{m_1 u_1}{m_1 + m_2}$$

$$\therefore \quad \bar{u}_1' = \frac{m_2 u_1}{m_1 + m_2} \qquad u_2' = \frac{m_1 u_1}{m_1 + m_2}.$$

Thus before collision, the observer at centre of mass., seem to approch both the particles to the observer.

After collision, the velocity $\vec{v}'$ of the combined mass in the centre of mass frame is given by conservation of momentum

$$(m_1+m_2)v' = m_1\vec{u}_1' + m_2\vec{u}_2'$$

$$= m_1\frac{m_2\vec{u}_1}{m_1+m_2} - m_2\frac{m_1\vec{u}_1}{m_1+m_2}$$

$$= 0.$$

So $v' = 0$

That is for an observer at centre of mass frame the combined system is at rest. So the kinetic energy after collision is zero. So in inelastic collision in centre of mass frame the original kinetic energy entirely converted into other energy (sound, heat etc)

Solve the Problem

Two bodies of mass 2 kg and 10 kg have position vector $(3\hat{i}+2\hat{j}-\hat{k})$ and $(\hat{i}-\hat{j}+3\hat{k})$ respectively. Find the position vector and the distance of centre of mass from origin.

Hints: The position of centre of mass $\vec{R} = \frac{m_1\vec{r}_1+m_2\vec{r}_2}{m_1+m_2}$

here $\vec{r}_1 = 3\hat{i}+2\hat{j}-\hat{k}$ $\vec{r}_2 = \hat{i}-\hat{j}+3\hat{k}$

Distance of the CM from origin

$$|\vec{R}| = \sqrt{x^2+y^2+z^2}$$
