Collision (Scattering) ①

Impulse : Impulse is the integral of force $\vec{F}$, over a time interval dt, for which it acts.

Let us consider a particle of mass m moving with velocity $\vec{v}$ under an external force $\vec{F}$. From Newton's 2nd law of motion we have

$$\vec{F} = \frac{d}{dt}(m\vec{v}) \longrightarrow ①$$

If the force $\vec{F}$ acts on the particle for a time interval dt then $\vec{F}dt$ is called impulse of the force $\vec{F}$. Since $\vec{F}$ is a vector quantity impulse of a force is also a vector quantity. The impulse of a force acting during time interval t_1 and t_2 is given by

$$\int_{t_1}^{t_2} \vec{F}\,dt = \int_{v_1}^{v_2} d(mv)$$

$$\vec{F}.\Delta t = \vec{F}(t_2 - t_1) = m\vec{v}_2 - m\vec{v}_1 \longrightarrow ②$$

which shows that impulse of a force is equal to the change in momentum of the particle where the force $\vec{F}$ acts.

The impulse of a force is a very big force which acts for a very short time so that the product $\vec{F}.\Delta t$ is a finite value.

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Scattering (Collision): When two particles approaches each other a force comes into play in between them for a finite time. As a result of this the velocity, momentum and kinetic energy of the particles are changed. This phenomenon is called as collision or scattering. There are two types of scatterings — (1) elastic scattering and (2) inelastic scattering.

Elastic Scattering: If the total kinetic energy and momentum of the two colliding particles before collision are equal to after collision then the scattering is called elastic scattering. In this scattering the two bodies may not be in physical contact.

Inelastic Scattering: In inelastic scattering the momentum of the colliding particles before collision and after collision are equal but kinetic energy before and after collision are not equal. Such type of collision are said to be perfectly inelastic when the particlecles stick together on impact and loss of kinetic energy becomes maximum. There are two types of inelastic collisions: (1) Endoergic collision

(2) exoergic "

Endoergic collision: A collision in which the kinetic energy of the final particles is ~~more~~ less than the initial kinetic energy of the particles is known as endoergic collision.

Exoergic collision: A collision in which the kinetic energy of the final particles is more than the kinetic energy of the initial particles is known exoergic collision.

Frame of reference: To study the collision between two particles we need a frame of reference. There are two types of reference frame or system.

1. Lab system or laboratory frame of reference.
2. Centre of mass system or frame of reference.

Laboratory frame: If the origin of the reference system is a point rigidly fixed to the laboratory then the system is called laboratory system or frame of reference.

Centre of mass frame or system: If the origin of the reference frame is a point fixed rigidly to the centre of mass of a system of particles on which no external force is acting is called centre of mass system (frame of reference)

In the centre of mass frame of reference the position vector of the centre of mass is $\vec{R}_c = 0$ as the centre of mass itself is origin of the refrence system :

∴ The velocity of the centre of mass $\frac{d\vec{R}}{dt} = 0 = \vec{V}$

∴ the linear momentum of the system $\vec{P} = m\vec{V} = 0$

Hence it is known as zero momentum frame.

In absence of any external force, the velocity of the centre of mass is constant. So centre of mass frame moves with a constant velocity with respect to laboratory frame. Hence centre of mass frame of reference is an inertial frame.

Elastic collision or scattering :

Laboratory and Centre of mass system :

In general, when the velocities of the two particles are very small, both the particles moves in such a direction that they come closer and collision takes place. fig(1). Here K_1 and K_2 are initial momenta of the two particles and P_1 and P_2 are final momenta of the two particles.

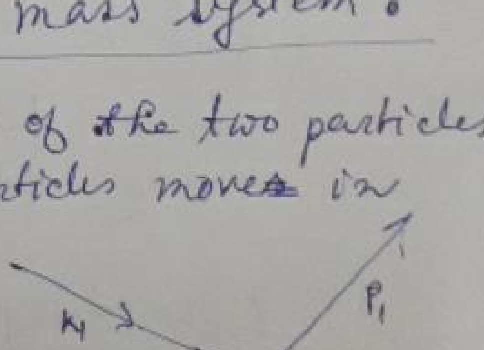

fig (1)

If one of the particles are at rest in the laboratory and other particle approaches it and collision takes place, then the setup is called laboratory frame (lab frame). fig.2

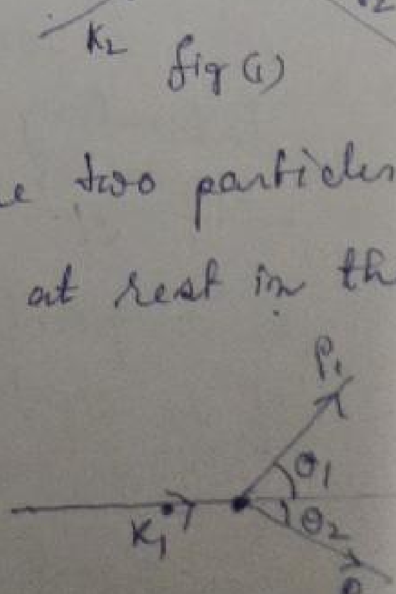

fig.2

In Centre of mass system, the collision of the particles is treated as if they are moving with equal and opposite momenta initially. In this case, we can say that centre of momentum of the particles ~~ai~~ is fixed. If number of colliding particles are more than two, then in CM system vector sum of initial momenta of the particles $\sum k_i$ is zero. Then, by the law of conservation of ~~momenta~~ linear momentum, the vector sum of final momenta must also be zero.

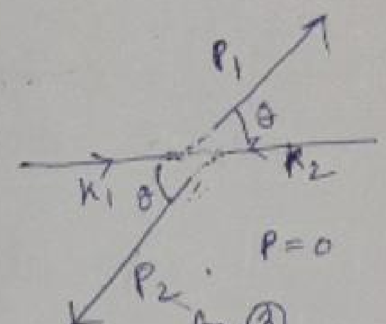

fig. ③

Elastic collision in one dimension:
(Laboratory system)

Let m_1 and m_2 be the masses of two particles and u_1, u_2 and v_1, v_2 are their respective velocities before and after collision along the line joining their centres.

Now according to the Principle of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \longrightarrow ①$$

and according to the law of conservation of ~~angular mo~~ kinetic energy

$$m_1 u_1^2 + m_2 u_2^2 = m_1 v_1^2 + m_2 v_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \longrightarrow ②$$

Dividing equ^n ② by ① we get

$$u_1 + v_1 = v_2 + u_2$$

$$u_1 - u_2 = v_2 - v_1 = -(v_1 - v_2) \longrightarrow (3)$$

Equ^n (3) shows that the relative velocity with which the two particles approach each other is equal to the relative velocity with which they recede from each other.

Velocity after collision

From equ^n (3) we have

$$v_1 = v_2 + u_2 - u_1$$

and $v_2 = v_1 + u_1 - u_2$

Substituting the value of v_2 in equ^n (1) we get

$$m_1(u_1 - v_1) = m_2(v_1 + u_1 - u_2 - u_2)$$

$$v_1(m_1 + m_2) = (m_1 - m_2)u_1 + 2m_2u_2.$$

$$\therefore v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2} \rightarrow (4)$$

similarly $$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_2 \rightarrow (5)$$

Special Cases:

(1) When $m_1 = m_2$ we have from equ^n (1)

$$u_1 - v_1 = v_2 - u_2$$

Again from equ^n (4) & (5) we get

$$v_1 = u_2 \quad \text{and} \quad v_2 = u_1$$

It shows that in one dimensional elastic collision of two particles of equal masses, the particles simply exchange their velocities as a result of their collision.

(2) When one body is initially at rest then $u_2 = 0$, so the final velocity

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1$$

and $$v_2 = \frac{2m_1}{m_1 + m_2} u_1$$

Now in this case we consider three situation.

(a) If $m_1 = m_2$ then $v_1 = 0$ and $v_2 = u_1$

i.e the 1st body stopped and the 2nd body takes off the velocity of 1st one.

(b) If $m_2 >> m_1$ ~~then~~ and $u_2 = 0$ then in comparison to mass m_2, $m_1 = 0$, $m_1 - m_2 = -m_2$

$m_1 + m_2 = m_2$ $\therefore$ $v_1 = -u_1$ and $v_2 = 0$.

This shows that when a very light particle collides a very heavier (massive) particle at rest, the heavy particle continues to remain at rest and the lighter particle reverses.

(c) If $m_1 >> m_2$ and $u_2 = 0$ then in comparison to mass m_1, $m_2 = 0$, $m_1 - m_2 = m_1$

$m_1 + m_2 = m_1$, hence $v_1 = u_1$ and $v_2 = 2u_1$

This shows that the velocity of the heavy particle remains almost the same after collision but velocity of the light particle becomes twice the velocity of the heavy particle.

: Collision in one dimension :

Centre of mass frame

When no external force is acting the velocity of the centre of mass is given by

$$\vec{V}_{cm} = \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1+m_2}$$

Velocity of the particle of mass m_1 before collision relative to the centre of mass frame according to Galelian transformation is given by.

$$\vec{u}_1' = \vec{u}_1 - \vec{V}_{cm} = \vec{u}_1 - \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1+m_2}.$$

$$= \frac{\cancel{m_1\vec{u}_1} + m_2\vec{u}_1 - \cancel{m_1\vec{u}_1} + m_2\vec{u}_2}{m_1+m_2}$$

$$u_1' = \frac{m_2(u_1-u_2)}{m_1+m_2}. \longrightarrow (6)$$

Velocity of the particle of mass m_2 before collision relative to the COM frame according to Galelian transformation.

$$\vec{u}_2' = \vec{u}_2 - \vec{V}_{cm} = \vec{u}_2 - \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1+m_2}$$

$$\vec{u}_2' = \frac{m_1(\vec{u}_2 - \vec{u}_1)}{m_1+m_2} \longrightarrow (7)$$

Similarly velocity of mass m_1 after collision relative to COM frame according to Galelian transformation is given by

$$\vec{v}_1' = \vec{v}_1 - \vec{v}_{cm} = \vec{v}_1 - \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1+m_2}$$

$$v_1' = \left(\frac{m_1-m_2}{m_1+m_2}\right)\vec{u}_1 + \frac{2m_2\vec{u}_2}{m_1+m_2} - \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1+m_2}$$

$$= \frac{\cancel{m_1\vec{u}_1} - m_2\vec{u}_1 + 2m_2\vec{u}_2 - \cancel{m_1\vec{u}_1} - m_2\vec{u}_2}{m_1+m_2}$$

$$v_1' = \frac{m_2(\vec{u}_1 - \vec{u}_2)}{m_1+m_2} \longrightarrow (8)$$

Similarly velocity of the particle of mass m_2 after collision in COM system applying Galelian transformation is given by

$$\vec{v}_2' = \vec{v}_2 - \vec{V}_{cm}$$

$$= \frac{(m_2-m_1)u_2 + 2m_1u_1}{m_1+m_2} - \frac{m_1u_1 + m_2u_2}{m_1+m_2}$$

$$* \quad = \frac{-m_1(u_2-u_1)}{m_1+m_2} \longrightarrow (9)$$

Thus in the centre of mass frame the incident and the target particles approach the centre of mass and then go away from the centre of mass in the opposite direction with the same velocities.

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Problem : (Please solve the problem)

Two particles each of mass 2 kg are moving with velocities $3\hat{i}+4\hat{j}$ m/s and $5\hat{i}+6\hat{j}$ m/s respectively. Find the kinetic energy of the system relative to centre of mass system.

Given $m_1 = m_2 = 2$ kg $\quad u_1 = 3\hat{i}+4\hat{j} \quad u_2 = 5\hat{i}+6\hat{j}$.

$$\vec{V}_{cm} = \frac{m_1u_1+m_2u_2}{m_1+m_2} =$$

Velocity of m_1 in Centre of mass frame

$$\vec{u}_1' = \vec{u}_1 - \vec{V}_{cm} =$$

Velocity of m_2 in COM frame

$$u_2' = \vec{u}_2 - \vec{V}_{cm} =$$

$\therefore$ K.E relative to COM system before collision =

$$= \frac{1}{2} m_1 u_1'^2 + \frac{1}{2} m_2 u_2'^2$$

$$=$$

Try to find the result yourself