Inverse Trigonometric Formulas (বিপৰীত ত্ৰিকোনোমিতীয় সূত্ৰ)

PART I

Functionsফলন)	**Domain(আদিক্ষেত্ৰ)**	**Range(পৰিসৰ)**
$\mathrm{Sin}^{-1} x$	[-1, 1]	[-π/2, π/2]
$\mathrm{Cos}^{-1}x$	[-1, 1]	[0, π]
$\mathrm{Tan}^{-1} x$	R	(-π/2, π/2)
$\mathrm{Cosec}^{-1} x$	R-(-1,1)	[-π/2, π/2]
$\mathrm{Sec}^{-1} x$	R-(-1,1)	[0,π]-{ π/2}
$\mathrm{Cot}^{-1} x$	R	[-π/2, π/2]-{0}

Inverse Trigonometric Formulas List (বিপৰীত ত্ৰকোনোমিতীয় সূত্ৰৰ তালিকা)

S.No	**Inverse Trigonometric Formulas (বিপৰীত ত্ৰকোনোমিতীয় সূত্ৰ)**
1	$\sin^{-1}(-x) = -\sin^{-1}(x), x \in [-1, 1]$
2	$\cos^{-1}(-x) = \pi - \cos^{-1}(x), x \in [-1, 1]$
3	$\tan^{-1}(-x) = -\tan^{-1}(x), x \in R$
4	$\mathrm{cosec}^{-1}(-x) = -\mathrm{cosec}^{-1}(x), \lvert x\rvert \geq 1$
5	$\sec^{-1}(-x) = \pi - \sec^{-1}(x), \lvert x\rvert \geq 1$
6	$\cot^{-1}(-x) = \pi - \cot^{-1}(x), x \in R$
7	$\sin^{-1}x + \cos^{-1}x = \pi/2 , x \in [-1, 1]$
8	$\tan^{-1}x + \cot^{-1}x = \pi/2 , x \in R$
9	$\sec^{-1}x + \mathrm{cosec}^{-1}x = \pi/2 , \lvert x\rvert \geq 1$

10	$\sin^{-1}(1/x) = \text{cosec}^{-1}(x)$, if $x \geq 1$ or $x \leq -1$
11	$\cos^{-1}(1/x) = \sec^{-1}(x)$, if $x \geq 1$ or $x \leq -1$
12	$\tan^{-1}(1/x) = \cot^{-1}(x), x > 0$
13	$\tan^{-1} x + \tan^{-1} y = \tan^{-1}((x+y)/(1-xy))$, if the value $xy < 1$
14	$\tan^{-1} x - \tan^{-1} y = \tan^{-1}((x-y)/(1+xy))$, if the value $xy > -1$
15	$2 \tan^{-1} x = \sin^{-1}(2x/(1+x^2)), \|x\| \leq 1$
16	$2\tan^{-1} x = \cos^{-1}((1-x^2)/(1+x^2)), x \geq 0$
17	$2\tan^{-1} x = \tan^{-1}(2x/(1-x^2)), -1<x<1$
18	$3\sin^{-1}x = \sin^{-1}(3x-4x^3)$
19	$3\cos^{-1}x = \cos^{-1}(4x^3-3x)$
20	$3\tan^{-1}x = \tan^{-1}((3x-x^3)/(1-3x^2))$
21	$\sin(\sin^{-1}(x)) = x, -1\leq x \leq1$
22	$\cos(\cos^{-1}(x)) = x, -1\leq x \leq1$
23	$\tan(\tan^{-1}(x)) = x, -\infty < x < \infty$.
24	$\text{cosec}(\text{cosec}^{-1}(x)) = x, -\infty < x \leq 1$ or $-1 \leq x < \infty$
25	$\sec(\sec^{-1}(x)) = x, -\infty < x \leq 1$ or $1 \leq x < \infty$
26	$\cot(\cot^{-1}(x)) = x, -\infty < x < \infty$.
27	$\sin^{-1}(\sin \theta) = \theta, -\pi/2 \leq \theta \leq\pi/2$
28	$\cos^{-1}(\cos \theta) = \theta, 0 \leq \theta \leq \pi$
29	$\tan^{-1}(\tan \theta) = \theta, -\pi/2 < \theta < \pi/2$
30	$\text{cosec}^{-1}(\text{cosec}\ \theta) = \theta, -\pi/2 \leq \theta < 0$ or $0 < \theta \leq \pi/2$
31	$\sec^{-1}(\sec \theta) = \theta, 0 \leq \theta \leq \pi/2$ or $\pi/2< \theta \leq \pi$

32	$\cot^{-1}(\cot\theta) = \theta, 0 < \theta < \pi$
33	$sin^{-1}x + sin^{-1}y = sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$ if x,y≥ 0 $and\ x^2 + +y^2 \leq 1$
34	$sin^{-1}x + sin^{-1}y = \pi - sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$ if x,y≥ 0 $and\ x^2 + y^2 > 1$
35	$sin^{-1}x + sin^{-1}y = -\pi - sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$ if xy$\geq$ $0\ and\ x^2 + +y^2 \leq 1$
36	$cos^{-1}x + cos^{-1}y = cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}$ if x,y≥ 0 $and\ x^2 + +y^2 \leq 1$
37	$cos^{-1}x + cos^{-1}y = \pi - cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}$ if x,y >0 $and\ x^2 + y^2 > 1$
37	$tan^{-1}x + tan^{-1}y = tan^{-1}\frac{x+y}{1-xy}$ if x > 0, y > 0 and xy < 1.
38	$tan^{-1}x + tan^{-1}y = \pi + tan^{-1}\frac{x+y}{1-xy}$ if x > 0, y > 0 and xy > 1.
39	$tan^{-1}x + tan^{-1}y = -\pi + tan^{-1}\frac{x+y}{1-xy}$ if x < 0, y > 0 and xy > 1.
40	$tan^{-1}x + tan^{-1}y + tan^{-1}z = tan^{-1}(\frac{x+y+z-xyz}{1-xy-yz-zx})$
41	$2sin^{-1}x = sin^{-1}(2x\sqrt{1-x^2})$
42	$2cos^{-1}x = cos^{-1}(2x^2\text{-}1)$
43	$2tan^{-1}x = tan^{-1}(\frac{2x}{1-x^2}) = sin^{-1}(\frac{2x}{1+x^2}) = cos^{-1}(\frac{1-x^2}{1+x^2})$
44	$3sin^{-1}x = sin^{-1}(3x - 4x^3)$
45	$3cos^{-1}x = cos^{-1}(4x^3 - 3x)$
46	$3tan^{-1}x = tan^{-1}(\frac{3x-x^3}{1-3x^2})$

We Know that if f:$X \to Y$ is one to and onto , then the inverse function

f^{-1}: Y→ X exists and is defined by

$f^{-1}(y) = x \ if \ f(x) = y$----------------(1)

As trigonometric functions are periodic, there are infinitely many values, whose image is same.

For example, sin($\frac{\pi}{4}$)=$\frac{1}{\sqrt{2}}$ and sin($\frac{3\pi}{4}$)= $\frac{1}{\sqrt{2}}$

Thus sine function y= f(x)= sinx is many one.

For each y∈ $[-1,1]$

There exist a unique number x in in each of the following intervals:

[- $\frac{3\pi}{2}$, $\frac{-\pi}{2}$] , [- $\frac{\pi}{2}$, $\frac{\pi}{2}$] , [$\frac{\pi}{2}$, $\frac{3\pi}{2}$]……………..

Such that y=sinx

If we restrict the domain of sine function to [- $\frac{\pi}{2}$, $\frac{\pi}{2}$] i.e if

Sin: [- $\frac{\pi}{2}$, $\frac{\pi}{2}$] →[-1,1]………………..(2)

Then y=sinx, x∈ [- $\frac{\pi}{2}$, $\frac{\pi}{2}$] is one-one and onto.

Thus its inverse is given by

sin^{-1} : [-1,1] → [- $\frac{\pi}{2}$, $\frac{\pi}{2}$]………………………….(3)

Hence sin^{-1} is a function and is defined by

Y= sin^{-1}x ≪=≫ siny=x

The function defined by (3) is called inverse sine function.

The elements in [- $\frac{\pi}{2}$, $\frac{\pi}{2}$] are called principal values of sin^{-1}.

From the definition of inverse functions, it follows that sin(sin^{-1}x)=x if -1≤ $x \leq 1$

And sin^{-1}(sinx)=x, if - $\frac{\pi}{2}$ ≤x ≤ $\frac{\pi}{2}$

Note : (i) sin^{-1}x ≠ $(sinx)^{-1}$, because $(sinx)^{-1}$=$\frac{1}{sinx}$

(ii) sin^{-1}x stands for an angle , but sinx is a number

1.Prove that (প্ৰমাণ কৰা যে)

(i) sin (sin^{-1}x)= x, x∈ [-1,1] (ii) $sin^{-1}(\sin\theta) = \theta$, $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

(iii) cos(cos^{-1}x)= x, x∈ [-1,1] (iv) $cos^{-1}(\cos\theta) = \theta$, $\theta \in [0, \pi]$

(v) tan (tan^{-1}x)= x, x∈ R (vi) $tan^{-1}(\tan\theta) = \theta$, $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

(vii) $cot(cot^{-1}$x) =x , x∈ R (viii) $cot^{-1}(\cot\theta) = \theta$, $\theta \in [0, \pi]$

(ix) cosec($cosec^{-1}$x)= x, x∈R- (-1,1) (x) $cosec^{-1}(\text{cosec}\theta) = \theta$,$\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$-{0}

(xi) sec(sec^{-1}x)= x, x∈ R-(-1,1) (xii) $sec^{-1}(\sec\theta) = \theta$, $\theta \in [0, \pi] - \{\frac{\pi}{2}\}$

Proof (প্ৰমাণ): (i) Let sin^{-1}x = θ , x∈ [-1,1] (ii) Let sin θ =x , $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

=> sin θ = x => sin^{-1}x= θ

=> sin (sin^{-1}x)= x => sin^{-1}(sin θ)= θ

2.Prove that (প্ৰমাণ কৰা যে)

(i) $sin^{-1}\frac{1}{x} = cosec^{-1}$x , x≥ 1 $or\ x \leq -1$ $(ii) cos^{-1}\frac{1}{x} = sec^{-1}$x , x≥ 1 $or\ x \leq -1$

(ii) $tan^{-1}\frac{1}{x} = cot^{-1}$x , x>0

Proof (প্ৰমাণ): (i) As (যিহেতু) $|x| \geq 1,\ x \neq 0$, $\left|\frac{1}{x}\right| \leq 1$,

$sin^{-1}\frac{1}{x}$ is defined ($sin^{-1}\frac{1}{x}$ সজ্ঞাবদ্ধ)

Let $cosec^{-1}$x = θ => cosec θ=x

So sin $\theta = \frac{1}{cosec\theta} = \frac{1}{x}$

=> $\theta = sin^{-1}\frac{1}{x}$

=> $sin^{-1}\frac{1}{x} = cosec^{-1}$x

3.Prove that (প্ৰমাণ কৰা যে)

(i) sin^{-1}(-x)= - sin^{-1}x, x∈ [-1,1] (ii) $cos^{-1}(-x) = \pi - cos^{-1}x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

(iii) tan^{-1}(-x)= - tan^{-1}x, x∈ R (iv) $cot^{-1}(-x) = \pi - cot^{-1}x \in R$

(v) $cosec^{-1}(-x) = cosec^{-1}x, |x| \geq 1$ (vi) $sec^{-1}(-x) = \pi - sec^{-1}x, |x| \geq 1$

Proof (প্ৰমাণ): (i) Let $sin^{-1}(-x) = \theta$

=> $\sin\theta = -x$

=> $-\sin\theta = x$

=> $\sin(-\theta) = x$

=> $-\theta = sin^{-1}x$

=> $\theta = -sin^{-1}x$

=> $sin^{-1}(-x) = -sin^{-1}x$

(ii) Let $cos^{-1}(-x) = \theta$

=> $\cos\theta = -x$

=> $-\cos\theta = x$

=> $\cos(\pi - \theta) = x$

=> $\pi - \theta = cos^{-1}x$

=> $\theta = \pi - cos^{-1}x$

=> $cos^{-1}(-x) = \pi - cos^{-1}x$

4. Prove that (প্ৰমাণ কৰা যে) (i) $sin^{-1}(x) = cos^{-1}\sqrt{1-x^2}$

(ii) $sin^{-1}(x) = tan^{-1}(\frac{x}{\sqrt{1-x^2}})$

(iii) $sin^{-1}(x) = sec^{-1}(\frac{1}{\sqrt{1-x^2}})$

Proof (প্ৰমাণ): (i) Let $sin^{-1}(x) = \theta$

=> $\sin\theta = x$

So $\cos\theta = \sqrt{1-sin^2\theta}$

=> $\cos\theta = \sqrt{1-x^2}$

=> $\theta = cos^{-1}(\sqrt{1-x^2})$

=> $sin^{-1}(x) = cos^{-1}(\sqrt{1-x^2})$

PART I