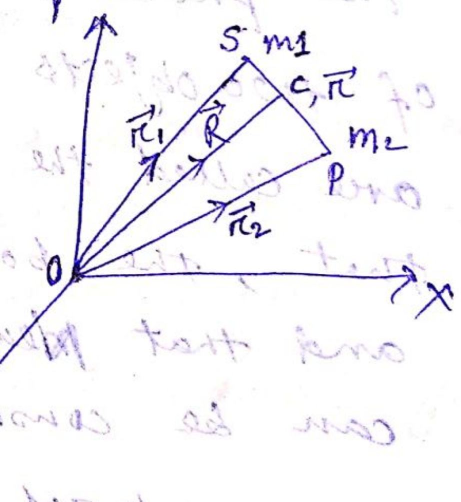
The motion of the centre of masses.

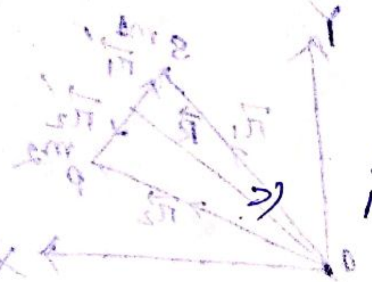
Let us consider an inertial ~~body~~ system having its origin at O, in which Newton's Law of motion holds true.



Let the bodies have masses m_1 and m_2 whose position vector relative to O are $\vec{r}_1$ and $\vec{r}_2$ respectively, at a particular instant of time. Let $\vec{R}$ be the position vector of the centre of mass of the pair, let $\vec{r}$ be the position vector of m_2 relative to m_1, i.e. $\vec{r} = \vec{r}_2 - \vec{r}_1$ —(1)

The equation of motion of S is

$$m_1 \ddot{\vec{r_1}} = \frac{G m_1 m_2}{r^2} \cdot \hat{r}$$



$$= \frac{G m_1 m_2}{r^2} \cdot \frac{\vec{r}}{r}$$

$$\Rightarrow \ddot{\vec{r_1}} = \frac{G m_2}{r^3} \cdot \vec{r} \longrightarrow (2)$$

The equation of motion of p is

$$m_2 \ddot{\vec{r_2}} = \frac{G m_1 m_2}{r^2} (-\hat{r})$$

$$= -\frac{G m_1 m_2}{r^3} \cdot \vec{r}$$

$$\Rightarrow \ddot{\vec{r_2}} = -\frac{G m_1}{r^3} \vec{r} \longrightarrow (3)$$

Now $\vec{R} = \dfrac{m_1\vec{r_1} + m_2 \vec{r_2}}{m_1+m_2}$

$$\Rightarrow (m_1+m_2)\vec{R} = m_1\vec{r_1} + m_2\vec{r_2}$$

$$\Rightarrow (m_1+m_2)\ddot{\vec{R}} = m_1\ddot{\vec{r_1}} + m_2\ddot{\vec{r_2}}$$

$$= \frac{G m_1 m_2}{r^3}\vec{r} - \frac{G m_1 m_2}{r^3}\vec{r}$$

[using (2) and (3)]

$$\Rightarrow \ddot{\vec{R}} = 0$$

$$\Rightarrow \vec{R} = \vec{c_1} + \vec{c_2} \longrightarrow (4)$$

This shows that the centre of mass of the system moves uniformly in a straight line in space.