Homogeneous liquids:—

We know that the pressure at a pt. given by

$$dp = \rho(X\,dx + Y\,dy + Z\,dz) \longrightarrow (i)$$

where ρ is the density of fluid at the point.

and, X, Y, Z are the component forces per unit mass.

Now if a liquid is homogeneous ρ is constant.

Thus from (i) it follows that ~~$X\,dx + Y\,d$~~

$$X\,dx + Y\,dy + Z\,dz = \frac{1}{\rho}\,dp$$

$$= d\left(\frac{p}{\rho}\right) \longrightarrow (ii)$$

i.e $X\,dx + Y\,dy + Z\,dz$ is a perfect differential.

But if $X\,dx + Y\,dy + Z\,dz$ is a perfect differential, then the system of forces is said to be conservative.

$\therefore$ A homogeneous liquid will be in equilibrium only when the system of forces is conservative.

Thus in this case we may lie,

$$X\,dx + Y\,dy + Z\,dz = -dv \quad (v \text{ being potential}$$

or, $\frac{dP}{\rho} = -dV$ (from (ii))

Integrating $\frac{p}{\rho} + V = c$, (c is constant of integration)

Note:— When ρ = constant, i.e, when the liquid is homogeneous the conditions of equilibrium can be written as.

$$\frac{\partial Z}{\partial y} = \frac{\partial Y}{\partial z}, \quad \frac{\partial X}{\partial z} = \frac{\partial Z}{\partial x}, \quad \frac{\partial X}{\partial y} = \frac{\partial Y}{\partial x} \qquad (**)$$

Heterogeneous liquid :—

In a heterogeneous fluid ρ ~~varish~~ varies, i.e, ρ is a. funcn of independent variable x, y, z. In this case. the system of forces X, Y, Z may maintain. equilibrium if

$$\frac{\partial}{\partial y}(\rho Z) = \frac{\partial}{\partial z}(\rho Y)$$

$$\frac{\partial}{\partial z}(\rho X) = \frac{\partial}{\partial x}(\rho Z)$$

$$\frac{\partial}{\partial x}(\rho Y) = \frac{\partial}{\partial y}(\rho X)$$

2019 Q: Show that a homogeneous liquid will be in equilibrium only when the system of forces conservation.