2014/2016 Q The surface of equal pressure are intersected orthogonally by the lines of force

Soln A surface of equal pressure,

say $p = \phi(x, y, z) =$ constant

and at any point $(x, y, z)$ of these surface, direction cosine of the normal are proportional to

$$\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}$$

ie proportional to $\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}$ as $\frac{\partial \phi}{\partial x} = \frac{\partial p}{\partial x}$

ie " " $\rho X, \rho Y, \rho Z$

ie " " $X, Y, Z$.

Hence the resultant force at any point is in the direction of the normal to the surface of equal pressure through the point.

So The surface of equal pressure are intersected orthogonally by the lines of force.

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★ Curves of equal ~~press~~ pressure & density :— ( § )

If a fluid is at ~~rest~~ under the forces X, Y, Z per a ~~lit mass~~ unit mass then find the diff$^n$ eq$^n$s of the curves of equal pressure & densities.

Let the fluid be heterogeneous (so that density is a func$^n$ of x, y, z) and incompressible.

Then the surface of equal pressure is given by equal pressure, ~~i.e.,~~ p = const i.e., $\partial p = 0$.

or, $X\,dx + Y\,dy + Z\,dz = 0 \rightarrow (1)$

& surfaces of equal density are given by $\rho = \text{const}$.

$\therefore\ d\rho = 0$

$$\Rightarrow \frac{\partial \rho}{\partial x}dx + \frac{\partial \rho}{\partial y}dy + \frac{\partial \rho}{\partial z}dz = 0 \rightarrow (2)$$

Curves of intersection of ~~eq~~ (1) & (2) are the curves of equal pressure & densities

from (1) & (2)

$$\frac{dx}{Z\frac{\partial \rho}{\partial y} - Y\frac{\partial \rho}{\partial z}} = \frac{dy}{X\frac{\partial \rho}{\partial z} - Z\frac{\partial \rho}{\partial x}} = \frac{dz}{Y\frac{\partial \rho}{\partial x} - X\frac{\partial \rho}{\partial y}} \rightarrow (3)$$

But from ~~on~~ the condition of equilibrium we have

$$Z\frac{\partial \rho}{\partial y} - Y\frac{\partial \rho}{\partial z} = \rho\left(\frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y}\right)$$

$$X\frac{\partial \rho}{\partial z} - Z\frac{\partial \rho}{\partial x} = \rho\left(\frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z}\right)$$

$$Y\frac{\partial \rho}{\partial x} - X\frac{\partial \rho}{\partial y} = \rho\left(\frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x}\right)$$

$$\frac{\partial^2 p}{\partial y \partial z} = \frac{\partial^2 p}{\partial z \partial y} \Rightarrow \frac{\partial}{\partial y}\left(\frac{\partial p}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{\partial p}{\partial y}\right)$$

With the help of these ~~dir~~ condition (3) becomes

$$\frac{dx}{\frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y}} = \frac{dy}{\frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z}} = \frac{dz}{\frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x}}$$

which is the required Equation of Equal Pressure and Equal Density

Q A fluid rest in equilibrium in a field of force $X = y^2+z^2-xy-xz$, $Y = z^2+x^2-zy-xy$, $Z = x^2+y^2-xz-yz$, show that the curve of equipressure and equidensity area set of circle.

Sol$^n$ The curves of equidensity and equipressure are given by

$$\frac{dx}{\frac{\partial Y}{\partial z}-\frac{\partial Z}{\partial y}} = \frac{dy}{\frac{\partial Z}{\partial x}-\frac{\partial X}{\partial z}} = \frac{dz}{\frac{\partial X}{\partial y}-\frac{\partial Y}{\partial x}}$$

$$\Rightarrow \frac{dx}{2z-y-2y+z} = \frac{dy}{2x-z-2z+x} = \frac{dz}{2y-x-2x+y}$$

$$\Rightarrow \frac{dx}{3(z-y)} = \frac{dy}{3(x-z)} = \frac{dz}{3(y-x)} = \frac{dx+dy+dz}{0} = \frac{xdx+ydy+zdz}{0}$$

$\therefore \quad dx+dy+dz=0 \longrightarrow (A)$

and $\quad xdx+ydy+zdz=0 \longrightarrow (B)$

Now Integrating (A) $\Rightarrow x+y+z=c_1 \longrightarrow (A')$

(B) $\Rightarrow \frac{x^2}{2}+y^2/2+z^2/2 = c_2 \longrightarrow (B')$

The curves of equipressure and equidensity are given by (A') and (B') togethere gives a set of circle.