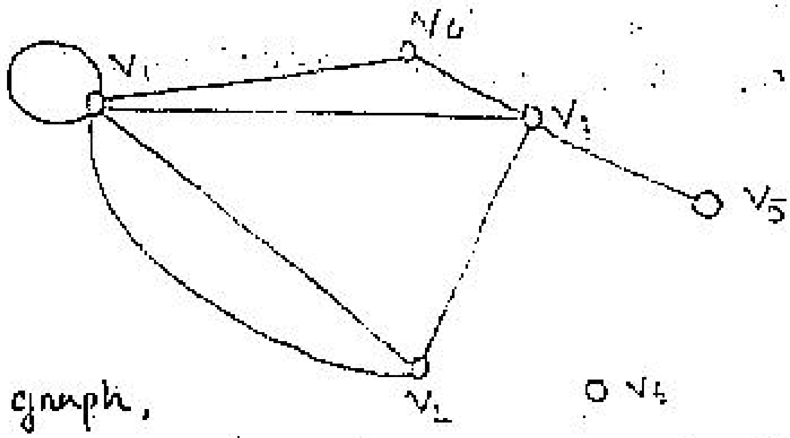
Degree of a vertex:

The number of edges incident on a vertex (with self loops counted twice) is called the degree of the vertex. The degree of a vertex v is denoted by $\deg(v)$ or $d(v)$.

Let us consider the following graph:



In the graph,

$\deg(v_1) = 6$ (we have counted twice for the loop*)

$\deg(v_2) = 3$

$\deg(v_3) = 4$

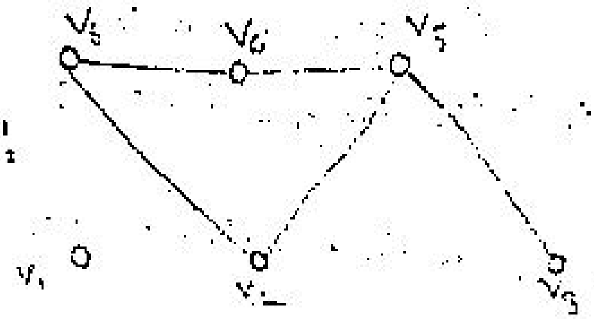
$\deg(v_4) = 2$

$\deg(v_5) = 1$

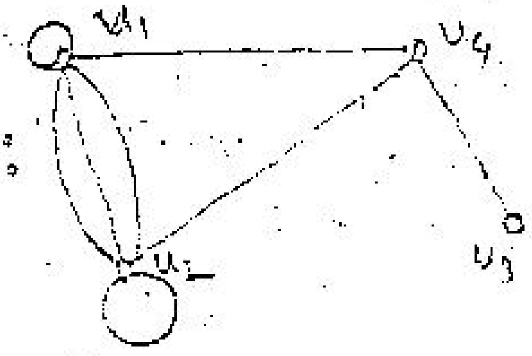
$\deg(v_6) = 0$

Question: Find the degrees of vertices in the following graphs:

G:



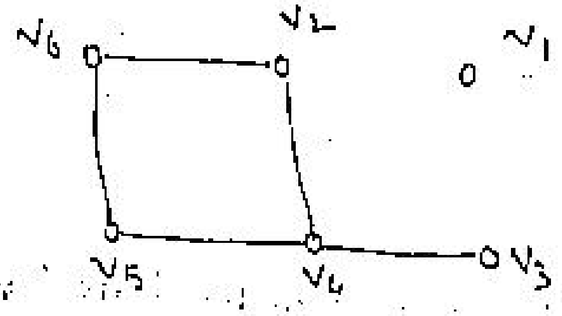
H:



Isolated and pendent vertices:

A vertex having no incident edges is called an isolated vertex. Therefore, degree of an isolated vertex is zero. A vertex of degree one is called a pendent vertex or end vertex.

In the following example. v_1 is an isolated vertex and v_3 is a pendent vertex.



Theorem: The sum of the degrees of the points (vertices) of a graph G is twice the number of edges in G, i.e

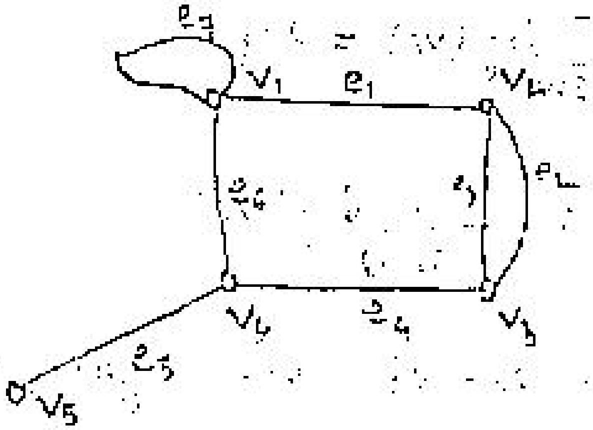
$$\sum \deg(v_i) = 2q, \text{ where } q \text{ is the number of edges in } G.$$

Pf: Consider a graph G. Since every line is incident with two points, it contributes 2 to the sum of the degrees of the vertices. Therefore sum of the degrees of all vertices in G is twice the number of edges in G. That is.

$$\sum \deg(v_i) = 2q, \ q \text{ is the number of edges}$$

Note: The above theorem is also called Handshaking theorem because if several people shake hands, then the total number of hands shaking must be even, because just two hands are involved in each handshake. ~~Also~~ From the above theorem we may conclude that the sum of the degrees of all vertices in a graph is even.

Now, let us consider the following graph:



Here total number of edges is 7

Now, $deg(v_1) + deg(v_2) + deg(v_3) + deg(v_4) + deg(v_5)$

$= 4 + 3 + 3 + 3 + 1$

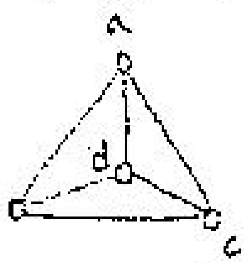
$= 14$

$= 2 \times 7$

$= 2 \times (\text{total number of edges})$

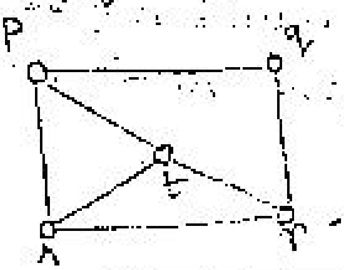
Question: verify Handshaking theorem for the following graph

(i)

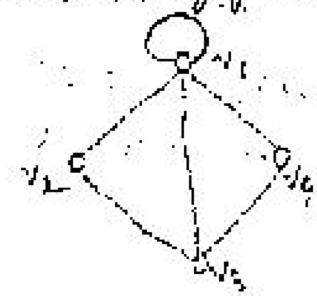


,

(ii)



(iii)



Theorem: The number of vertices of odd degree in a graph G is always even.

Pf: Let $G = (V, E)$ be a graph. Let U denote the set of even degree vertices and W denote the set of odd degree vertices. Since sum of the degrees of all vertices in G is twice the number of edges

$\therefore \sum_{v_i \in V} \deg(v_i) = 2q$, where '$q$' is the number of edges.

$$\Rightarrow \sum_{v_i \in U} \deg(v_i) + \sum_{v_i \in W} \deg(v_i) = 2q$$

$$\Rightarrow \sum_{v_i \in W} \deg(v_i) = 2q - \sum_{v_i \in U} \deg(v_i)$$

Now, since U is the set of even degree vertices

$\therefore \sum_{v_i \in U} \deg(v_i)$ is even.

$\therefore 2q - \sum_{v_i \in U} \deg(v_i)$ is even.

$\therefore \sum_{v_i \in W} \deg(v_i)$ is even.

But $\deg(v_i)$ are even, $\forall v_i \in W$.

$\therefore$ In order, the sum $\sum_{v_i \in W} \deg(v_i)$ to be even, we must have the number ~~of~~ vertices in W is even.

i.e number of odd vertices is even.

<u>problem</u>: Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree 2. Also draw the graph.

<u>solution</u>: Suppose the graph has q number of edges. Given there are 6 vertices, 2 of degree 4 and 4 of degree 2.

Now, from the Handshaking theorem, we get

$$\sum_{i=1}^{6} \deg(v_i) = 2q$$

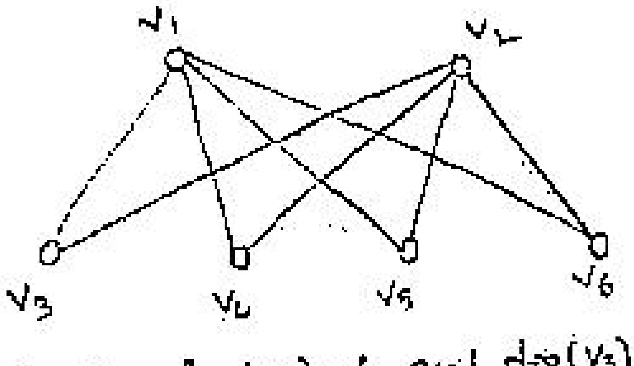
$$\Rightarrow \quad 4+4+2+2+2+2 = 2q$$

$$\Rightarrow \quad 16 = 2q$$

$$\Rightarrow \quad q = 8$$

$\therefore$ Number of edges in the graph is 8.

Such a graph is shown below:



Here $\deg(v_1) = 4$, $\deg(v_2) = 4$ and $\deg(v_3) = \deg(v_4) = \deg(v_5) = \deg(v_6) = 2$