$$\therefore \quad \nu' = \nu\left(1 - \frac{gd}{c^2}\right)$$

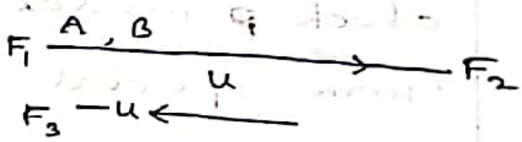
$$\Rightarrow \frac{1}{\nu'} = \frac{1}{\nu}\left(1 - \frac{gd}{c^2}\right)^{-1}$$

$$\Rightarrow T' = T\left(1 + \frac{gd}{c^2}\right).$$

Clock paradox:

Let us consider two identical clocks A and B originally together and at rest and let a force F be applied for a short time to the clock B giving it the velocity u with which it then travel away from A at a constant rate for a time which is long compared. With that necessary for the acceleration. At the end of this time let a second force F_2 be applied in the reverse direction, which brings B to rest and starts it back towards A with the reversed velocity $-u$.

Finally, when it has returned to the neighbourhood of A, let the clock B be brought to rest by the action of a third force F_3.



Since by hypothesis the time intervals necessary for the acceleration and deceleration of clock B are made negligibly short compound with the time of interval at velocity u, we can write in accordance with the decreased rate of a moving clock given by the special theory of relativity

$$\Delta t_A = \frac{\Delta t_B}{\sqrt{1-\frac{u^2}{c^2}}} = \Delta t_B\left(1+\frac{1}{2}\frac{u^2}{c^2}+\cdots\right) \longrightarrow (1)$$

$$\Rightarrow \Delta t_A > \Delta t_B.$$

Therefore the clock B will go slower than the clock A.

Reversing the process i.e. taking B at rest and A moving with velocity $-u$ and returning with velocity $+u$, i.e. taking A as the moving clock, it then seems as if A should be the clock that register the smaller number of division (i.e. A will go slower than B)

$$\Delta t_B = \frac{\Delta t_A}{\sqrt{1-\frac{u^2}{c^2}}} = \Delta t_A\left(1+\frac{1}{2}\frac{u^2}{c^2}+\cdots\right) \longrightarrow (2)$$

$$\therefore \Delta t_B > \Delta t_A$$

This ~~the~~ is the paradox and is called clock paradox. This called not be explained from special theory of relativity. This apparent paradox is however, readily solved with the help of the general theory of relativity. In the first case, clock B was subjected to forces at the beginning, when it was acceleration, then when its motion was changed from u to $-u$. and again when it was brought to rest from the velocity $-u$. To preserve the same state of attains when A is taken as moving and B is at rest, we must introduce a uniform gravitational field which will give A of the same type of motion as B had preciously and will keep B as rest. This is accordance with the principle of equivalence.

Let us take $\Delta t_A = T_A + T_A' + T_A'' + T_A''' \rightarrow (3)$

and $\Delta t_B = T_B + T_B' + T_B'' + T_B''' \longrightarrow (4)$

where T_A and T_B are the time measurement referred to the two clocks during which the clock A is now regarded as having the uniform velocity u and T_A', T_A'', T_A''' and T_B', T_B'', T_B''' are the times needed for their changes in the velocity of the clocks which are brought about at the beginning, middle and the end of the experiment by temporary indroduction of gravitational field as mentioned above.

Let us take the latter interval as very short compared with the time of movement of the clocks from beginning to end. Since the clock A is new the one which moves. We can write in accordance with the special theory of relativity,

$$T_B = \frac{T_A}{\sqrt{1-\frac{u^2}{c^2}}}$$

or $T_A = T_B\left(1 - \frac{1}{2}\frac{u^2}{c^2} + \cdots\right) \longrightarrow (5)$

This is in contrast to relation (1) when (B) was taken as the moving clock.

Since the law will be practically at the same potential when the gravitational field are introduced to the beginning and at the end of the experiment, we can write

$$T_A' = T_B' \quad \text{and} \quad T_A'' = T_B'' \longrightarrow (6)$$

On the otherhand, when the gravitational field is introduced at the middle of the ~~expree~~ experiment, the two clocks will be at great distance from each other and we have

$$T_A'' = T_B''\left(1 + \frac{gd}{c^2}\right) \longrightarrow (7)$$

Now 2d is the total disdance travelled at the speed u, then

$$2d = u \times T_B$$

$$\Rightarrow d = \frac{u \times T_B}{2}$$

Since 2u (i.e. u-(-u)) is the total change in velocity in time T_B'', so

$$g T_B'' = 2u$$

$$\Rightarrow g = \frac{2u}{T_B''}$$