

$$\Rightarrow \vec{r} \cdot \vec{u} = 0, \quad \left[ \vec{r} \cdot (\vec{r} \times \vec{v}) = \vec{r} \cdot \vec{r} \cdot \vec{v} \right]$$

$$\rightarrow \textcircled{12} \quad = 0$$

The eq<sup>n</sup>:  $\textcircled{12}$  represents a plane through 3 perpendicular to the vector  $\vec{h}$ .

$$\vec{h} = h_1 \hat{i} + h_2 \hat{j} + h_3 \hat{k}$$

Thus the orbit of the planet (or satellite) around the sun (or earth) is a plane curve.

### Solution of the Two-Body Problem

The eq<sup>n</sup> of motion of the two body problem is  $\ddot{\vec{r}} = -\frac{\mu \vec{r}}{r^3}$   $\textcircled{1}$

Taking vector product of equation with  $\vec{h}$ , we get

$$\vec{h} \times \ddot{\vec{r}} = -\frac{\mu}{r^3} (\vec{h} \times \vec{r})$$

$$= -\frac{\mu}{r^3} (\dot{\vec{r}} \times \vec{r}) \times \vec{r}$$

$$= -\frac{\mu}{r^3} [(\vec{r} \cdot \dot{\vec{r}}) \vec{r} - (\vec{r} \cdot \vec{r}) \dot{\vec{r}}] \quad \left[ \because \vec{h} = \vec{r} \times \dot{\vec{r}} \right]$$

$$0 = -\frac{\mu}{r^3} [r^2 \dot{\vec{r}} - (r\dot{r}) \vec{r}]$$

$$= -\mu \left( \frac{\dot{\vec{r}}}{r} - \frac{\vec{r} \cdot \dot{\vec{r}}}{r^2} \right) \quad [\because \vec{r} \cdot \dot{\vec{r}} = r\dot{r}]$$

$$\Rightarrow -\frac{d}{dt}(\vec{r} \times \dot{\vec{r}}) = -\mu \cdot \frac{d}{dt} \left( \frac{\vec{r}}{r} \right)$$

$$\Rightarrow \frac{d}{dt}(\vec{r} \times \dot{\vec{r}}) = \mu \frac{d}{dt} \left( \frac{\vec{r}}{r} \right)$$

Integrating,  $\vec{r} \times \dot{\vec{r}} = \mu \frac{\vec{r}}{r} + \vec{p}$ ,

where  $\vec{p}$  is the vector constant of integration,

Now taking the dot product of this Eqn with  $\vec{r}$ , we obtain the following

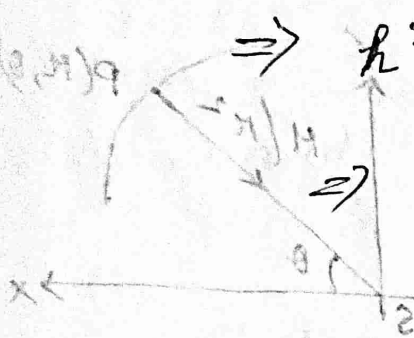
$$\text{eqn } \vec{r} \cdot (\vec{r} \times \dot{\vec{r}}) = \vec{r} \cdot \mu \frac{\vec{r}}{r} + \vec{r} \cdot \vec{p}$$

$$\Rightarrow (\vec{r} \times \dot{\vec{r}}) \cdot \vec{r} = \mu \frac{r^2}{r} + \vec{r} \cdot \vec{p}$$

$$\Rightarrow \vec{r} \cdot \dot{\vec{r}} = \mu + r p \cos \omega \quad [\because \vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c} \text{ and } \vec{a} \cdot \vec{a} = a^2]$$

where  $\alpha$  is the angle between the constant vector  $\vec{p}$  and the radius vector  $\vec{r}$

$$\Rightarrow h^2 = \mu r + r p \cos \alpha$$



$$\Rightarrow r = \frac{h^2}{\mu + p \cos \alpha}$$

which represent a conic with eccentricity.

$e = \frac{p}{\mu}$  and semi-latus rectum

$$l = \frac{h^2}{\mu}$$

It is an ellipse if  $e < 1$

a Hyperbola if  $e > 1$  and

(a) parabola if  $e = 1$

$$\frac{ab}{fb} \cdot \frac{1}{a} = \left(\frac{1}{a}\right) \frac{b}{fb} = \frac{ab}{fb}$$

$$\frac{ab}{fb} \frac{pb}{ab} \frac{1}{a} =$$