

Ex: Solve the system of eqⁿ.

$$2x_1 - x_2 + 0x_3 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$0 \cdot x_1 - x_2 + 2 \cdot x_3 = 1$$

Using Gauss - Seidel method take initial approximation as $x^{(0)} = 0$ and perform 3 iteration.

Sol → The given system of eqⁿ can be write

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad L = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(D+L) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{bmatrix}$$

Gauss seidel method gives .

$$X^{(k+1)} = -(D+L)^{-1} U X^{(k)} + (D+L)^{-1} b$$

$$\therefore (D+L)^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 1/4 & 1/2 & 0 \\ 1/8 & 1/4 & 1/2 \end{bmatrix}$$

$$(D+L)^{-1} b = \begin{bmatrix} 1/2 & 0 & 0 \\ 1/4 & 1/2 & 0 \\ 1/8 & 1/4 & 1/2 \end{bmatrix} \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7/2 \\ 9/4 \\ 13/8 \end{bmatrix}$$

$$(D+L)^{-1} U = \begin{bmatrix} 1/2 & 0 & 0 \\ 1/4 & 1/2 & 0 \\ 1/8 & 1/4 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1/2 & 0 \\ 0 & -1/4 & -1/2 \\ 0 & -1/8 & -1/4 \end{bmatrix}$$

$$\therefore X^{(k+1)} = -(D+L)^{-1} U X^{(k)} + (D+L)^{-1} b$$

$$= - \begin{bmatrix} 0 & -1/2 & 0 \\ 0 & -1/4 & -1/2 \\ 0 & -1/8 & -1/4 \end{bmatrix} X^{(k)} + \begin{bmatrix} 7/2 \\ 9/4 \\ 13/8 \end{bmatrix}$$

$$\text{Now, } X^{(0)} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

1st iteration

$$x^1 = -(D+L)^{-1} U x^{(0)} + (D+L)^{-1} b$$

$$= - \begin{bmatrix} 0 & -1/2 & 0 \\ 0 & -1/4 & -1/2 \\ 0 & -1/8 & -1/4 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 7/2 \\ 9/4 \\ 13/8 \end{bmatrix}$$

$$= \begin{bmatrix} 3.5 \\ 2.25 \\ 1.625 \end{bmatrix}$$

2nd iteration

$$x^2 = -(D+L)^{-1} U x^{(1)} + (D+L)^{-1} b$$

$$= - \begin{bmatrix} 0 & -1/2 & 0 \\ 0 & -1/4 & -1/2 \\ 0 & -1/8 & -1/4 \end{bmatrix} \begin{bmatrix} 3.5 \\ 2.25 \\ 1.625 \end{bmatrix} + \begin{bmatrix} 7/2 \\ 9/4 \\ 13/8 \end{bmatrix}$$

$$= \begin{bmatrix} 4.625 \\ 3.625 \\ 2.3125 \end{bmatrix}$$

3rd iteration

$$x^{(3)} = -(D+L)^{-1} U x^{(2)} + (D+L)^{-1} b$$

$$= - \begin{bmatrix} 0 & -1/2 & 0 \\ 0 & -1/4 & -1/2 \\ 0 & -1/8 & -1/4 \end{bmatrix} \begin{bmatrix} 4.625 \\ 3.625 \\ 2.3125 \end{bmatrix} + \begin{bmatrix} 7/2 \\ 9/4 \\ 13/8 \end{bmatrix}$$

$$= \begin{bmatrix} 3.3125 \\ 4.3125 \\ 2.6563 \end{bmatrix}$$