Using these results in (7), we get

$$T_A'' = T_B'' + T_B'' \frac{2u}{T_B''} \frac{u \times T_B}{2c^2}$$

$$\Rightarrow T_A'' = \left(T_B'' + \frac{u^2}{c^2} T_B\right) \longrightarrow (8)$$

Combining these equations (5), (6) and (8) with (3) and (4), we obtain

$$\Delta t_A = T_B\left(1 - \frac{1}{2}\frac{u^2}{c^2} + \ldots\ldots\right) + T_B' + T_B'' + \frac{u^2}{c^2} T_B + T_B'''$$

$$= T_B\left(1 + \frac{1}{2}\frac{u^2}{c^2} + \ldots\ldots\right) + T_B' + T_B'' + T_B'''$$

Since the primed quantities are very short compared with unprimed quantities (i.e. T_B)

$$\therefore \quad \Delta t_A = \Delta t_B \left(1 + \frac{1}{2}\frac{u^2}{c^2}\right) \longrightarrow (9)$$

Comparing this result (9) with the equation (1) we now see that whether we consider A or B to be the moving clock, we obtain the same expression.

The solution known as clock paradox of the special theory of relativity gives an example of the justification for all kinds of motion as relative, that has been possible by the ~~principle~~ adoption of principle of equivalence i.e. GTR.

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(*) imp. **Newton's equations of motion as an approximation of geodesic equations OR trajectory of a free particle:→**

Let us find the trajectory of a free particle in the space-time whose geometry is governed by the line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \longrightarrow (1)$$

Consider the motion of a test particle in the case of a weak static gravitational field. The motion of a test particle is governed by the geodesic equation

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0 \longrightarrow (2)$$

where $\Gamma^\mu_{\alpha\beta}$'s are christoffel symbols of the second kind and are given by

$$\Gamma^\mu_{\alpha\beta} = g^{\mu\nu} \Gamma_{\nu,\alpha\beta} \quad \text{and}$$

$$\Gamma_{\nu,\alpha\beta} = \frac{1}{2}\left(\frac{\partial g_{\nu\alpha}}{\partial x^\beta} + \frac{\partial g_{\nu\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\nu} \right)$$

Now in special theory of relativity, the line element corresponds to a Euclidian space time (flat space-time) and all $g_{\mu\nu}$'s are constant and independent of the coordinates.

Consequently all $\Gamma^\mu_{\alpha\beta}$ vanish and in this case equation (2) reduces to the equation of a straight line, i.e.

$$\frac{d^2x^\mu}{ds^2} = 0 \longrightarrow (3)$$

It is remarkable that the metric tensor $g_{\mu\nu}$ determines both the geometry of the space-time continuum and also the trajectory.

For Euclidian space, the components of the metric tensor are all constants and are given by

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \longrightarrow (4)$$

Since $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ is Riemannian space

$= -dx^2 - dy^2 - dz^2 + c^2 dt^2$ is Euclidian space.

Let us assume that the $g_{\mu\nu}$'s are not constants but differ from the values given by (4) by infinitesimal amount viz.. in a weak (static) gravitational field.

So, we can put

$$g_{\mu\nu} = \eta_{\mu\nu} + \psi_{\mu\nu} \longrightarrow (5)$$

where $\eta_{\mu\nu}$ is the flat-space-time metric coefficient and $|\psi_{\mu\nu}| << 1$ (i.e. $\psi_{\mu\nu}$ are small quantities) and

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