Electromagnetic Induction

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Faraday's Law of Electromagnetic Induction:

Michel Faraday observed that

If a magnet is moved about in a neighbourhood of a closed circuit of wire then a current will be induced in the wire so long as the movement lasts, but will disappear as soon as the movement is ceased.

The same effect is produced in the reverse case i.e. the circuit is moving and the magnet is kept static.

A transient current is introduced in a loop of wire when the current in another adjacent circuit is turned on and off. So a transient current is induced in a circuit when the flux linked through the circuit is changing.

This phenomenon is known as magnetic induction. Summing Faraday's observation the rule is obtained as follows,

When the magnetic flux through the circuit is changing, an electromotive force is induced in the circuit, the magnitude of which is proportional to the rate of change of flux.

If e is the e.m.f. and Ø be flux then

$$|e| \propto \frac{d\emptyset}{dt} \rightarrow (i)$$

The direction of the induced e.m.f. by the method , which is explained by the Lenz's Law.

Let a circuit of any shape ABC is moving in a time independent magnetic field *B* with a velocity v . Now considering an elementary section of *dl*. After time *t* it will move to P'Q' at distance v.*dt* from PQ. If the velocity of free electron relative to the circuit be u then the relative velocity to the field will be $v + u$. Therefore each electron will experience a force $[e((v + u) \times B)]$ The component of this force along *dl* is $[e((v + u) \times B)]i$ where$i$ is the unit vector in the direction of PQ.

Since i is parallel to u, therefore $u \times B.i = 0$

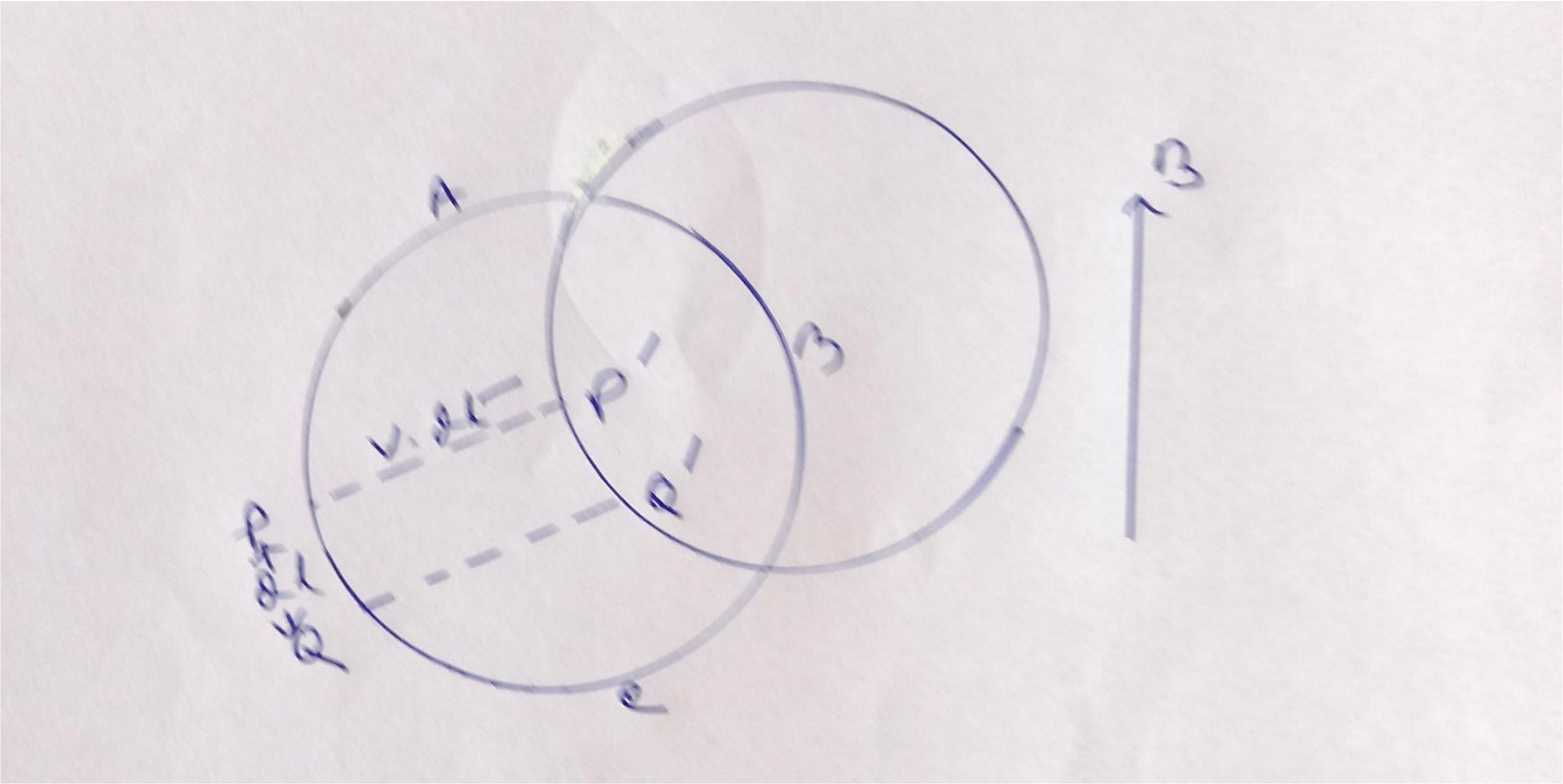

Fig 1

Hence

$[e(v+u)\times B]i = e.v\times B.i + e.u\times B.i = e.v\times B.i \rightarrow (ii)$

From the above equation it is clear that an electric field

$v\times B = E$ will be induced in the circuit, the component of which along the wire being $(v\times B).i.$ The $\mathrm{e.m.f.}$ induced in the circuit will be the line integral of the field E round the circuit. Therefore the induced $\mathrm{e.m.f.}$ is

$$e = \oint (v\times B) i dl \rightarrow (iii)$$

As the circuit is moving after time dt the elementary section dl will sweep out an area $PP'QQ' = v.dt \times i.dl$

Therefore the flux passing through the element is

$$(v.dt \times i.dl).B$$

The flux over the entire band is

$$d\phi = \oint B(v.dt \times i.dl) \rightarrow (iv)$$

$$\frac{d\phi}{dt} = \oint B(v \times i.dl) = -\oint (v \times B) i.dl \rightarrow (v)$$

Combining equation *(iii)* and *(v)* we get

$$e = -\frac{d\phi}{dt} \rightarrow (vi)$$

Integral & Differential Form of Faraday's Law:

The induced e.m.f. is equal to the line integral of the induced electric field *E* around the coil i.e.

$$e = \oint E.i.dl \rightarrow (vii)$$

And the flux through the coil is

$$\phi = \iint B.i.ds \rightarrow (viii)$$

Where *ds* = area PP'QQ' . Therefore equation (vi) can be written as

$$\oint E.i.dl = -\iint \frac{dB}{dt} i.ds \rightarrow (ix)$$

This equation (ix) is known as Integral Form of Faraday's law.

Using stock's theorem

$$\oint E.i.dl = \iint curl\ E.i.ds = -\iint \frac{\partial B}{\partial t}.ids \rightarrow (x)$$

Therefore

$$\iint \left(curl\ E + \frac{\partial B}{\partial t}\right) i.ds = \rightarrow (xi)$$

Here we replace the total time derivative by partial derivative because we consider only the change in the field B with time at the fixed position of the elementary area.

Now equation *(xi)* is true for any arbitrary area. Therefore we can write

$$curl\, E = \nabla \times E = -\frac{\partial B}{dt} \rightarrow (xii)$$

This equation is known as Differential Form of Faraday's Law. Now if we take divergence of equation (xii)

then we get

$$\nabla.(\nabla \times E) = \frac{\partial}{\partial t}(\nabla. B) = 0 \rightarrow (xiii)$$

Therefore $(\nabla. B\ )$ must be independent of time at every point. The above condition will be fulfilled if we assume

$$\nabla. B = 0 \rightarrow (xiv)$$

This means *B* is always solenoid. From equation *(xiii)* two important conclusion can be made -

First one is if magnetic field changes with time, the electric field no longer remains conservative

Second one is all magnetic poles in pairs positive and negative. Hence we can explain on the basis of Faraday's Laws, how the electric and magnetic fields are interrelated.

Lenz's Law:

This law states that The direction of the induced e.m.f. is such that the magnetic flux associated with the current generated by it, opposes the original change of flux causing the e.m.f.

As shown in Fig (ii), if a magnet is moved in the direction of arrow, the induced e.m.f. will flow in the direction such that its own flux will oppose the increase in the flux of the magnet.

Self Inductance and Mutual Inductance:

When a current flows in a circuit, a magnetic flux Ø is linked through the circuit due to its own field. This flux is proportional to the current I flowing through the circuit.

$$\phi = L.I \rightarrow (xv)$$

Where L is a constant.

Let magnetic flux with time, then

$$\frac{d\phi}{dt} = \frac{d\phi}{dI}.\frac{dI}{dt} = L\frac{dI}{dt} \rightarrow (xvi)$$

Here $L = \frac{d\phi}{dI}$

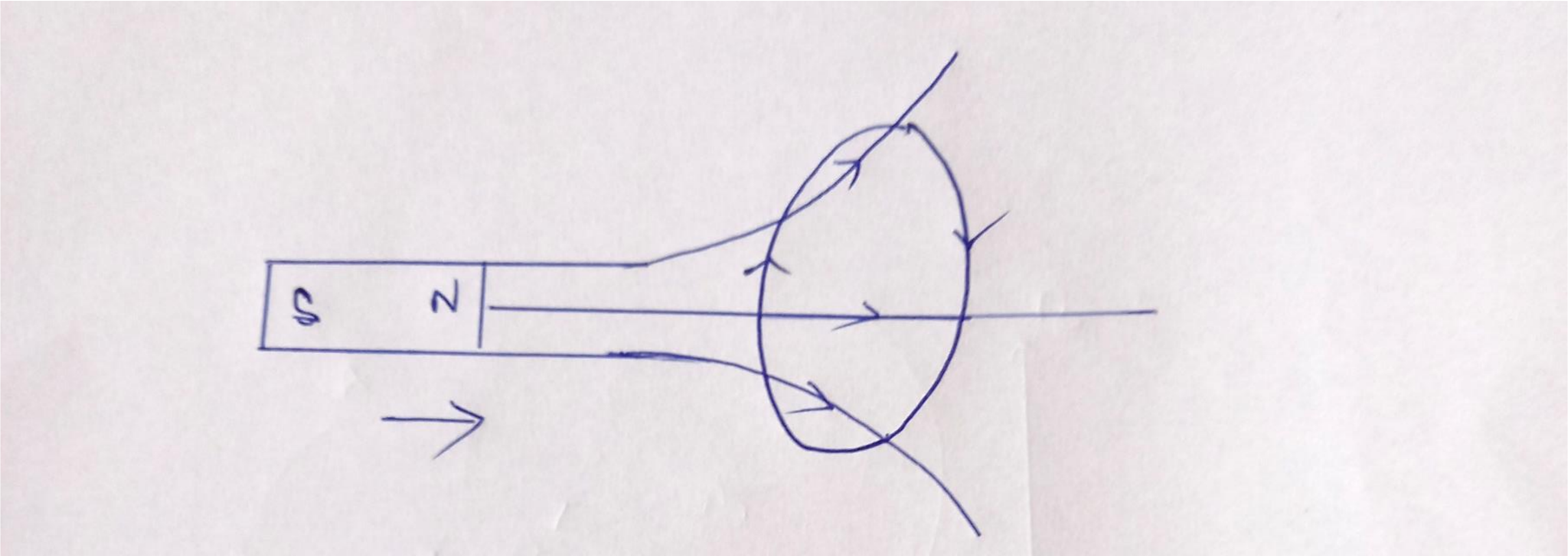

Fig 2

This quantity L is called self-inductance and it depends on the geometry of the circuit. So we can write that

$$e = -\frac{d\phi}{dt} = -L\frac{dI}{dt} \rightarrow (xvi)$$

Therefore we can define the self-inductance as the e.m.f. induced in the circuit, when the rate of change of current is unity.

The unit of self-inductance is Henry. It is defined as the seif-inductance of a coil when an e.m.f. of one volt is produced the change of current being I ampere per second. Therefore

$$1\, Henry = 10^9\, e.m.u.of\ self\ inductance$$

Now let a coil carries a current I_1 and second coil in which a current I_2 is flowing brought near to the first coil. Then there will be a linkage of flux through the first coil due to current flowing in the second.

$$\phi_1 = L_{12} I_2 \rightarrow (xvii)$$

Where L_{12} is a constant. Similarly there will be linkage of flux through the second coil due to current in the first.

$$\phi_2 = L_{21} i_1 \rightarrow (xviii)$$

Where L_{21} is a constant.

The potential energy of the system will be

$$U_p = -\phi_1 I_1 = -I_1 \int B_1 i_1 ds = -I_1 \int \nabla \times A_{12}. i_1 ds \rightarrow (xix)$$

Where A is a vector potential and $B = \nabla \times A$

Using Stock's Theorem we get

$$U_p = -I_1 \int A_{12} i_1 dl_1 \rightarrow (xx)$$

$$U_p = -I_1 \int \left[\frac{\mu}{4\pi} \int \frac{I_1 I_2 dl_2}{|r|} \right]. i_1 dl_1$$

$$U_p = -\frac{\mu}{4\pi} I_1 I_2 \iint \frac{i_1 dl_1. i_2 dl_2}{|r|} \rightarrow (xxi)$$

Again

$$U_p = -MI_1I_2 \rightarrow (xxii)$$

Comparing equation (xxi) and $(xxii)$ we get

$$M = \frac{\mu}{4\pi}\iint \frac{i_1 dl_1 i_2 dl_2}{|r|} \rightarrow (xxiii)$$

Where M is known as Mutual Inductance

This equation $(xxiii)$ is known as Neumann's formula.