Problem: How many vertices are needed to construct a graph with 6 edges in which each vertex is of degree 2.

Solution: Suppose there are p vertices in the graph each of degree 2.

$$\therefore \sum_{i=1}^{p} \deg(v_i) = 2\times q = 2\times 6 = 12$$

$$\Rightarrow \sum_{i=1}^{p} \deg(v_i) = 12$$

$$\Rightarrow \deg(v_1) + \deg(v_2) + \deg(v_3) + \cdots + \deg(v_6) = 12$$

$$\Rightarrow 2 + 2 + 2 + \cdots + 2 = 12$$

$$\Rightarrow p.2 = 12$$

$$\Rightarrow p = 6$$

$\therefore$ 6 vertices needed to construct the graph.

Problem: It is possible to construct a graph with 12 vertices such that two of the vertices have degree 3 and the remaining vertices have degree 4.

Soln: Suppose it is possible to construct a graph with 12 vertices out of which 2 of them are having degree 3 and remaining vertices are having degree 4.

$\therefore \sum_{i=1}^{12} \deg(v_i) = 2q$

$\Rightarrow 3+3+\underbrace{4+4+4+\cdots+4}_{10\text{-times}} = 2q$

$\Rightarrow 6+10\times 4 = 2q$

$\Rightarrow 46 = 2q$

$\Rightarrow q = 23$ (which is a non negative integer)

$\therefore$ It is possible to construct the graph with 23 edges and 12 vertices.

Problem: It is possible to draw a simple ~~graph~~ graph with 4-vertices and 7 edges? Justify.

Solution: In a simple graph with p-vertices, the maximum number of edges will be $\frac{p(p-1)}{2}$.

Hence a simple graph with 4 vertices will have atmost $\frac{4\times(4-1)}{2} = \frac{4\times 3}{2} = 6$ edges.

$\therefore$ A simple graph with 4 vertices cannot have 7 edges.

$\therefore$ It is not possible to draw a simple graph of 4-vertices and 7 edges.

Problem: Show that the maximum degree of any vertex in a simple graph with p- vertices is $(p-1)$.

Solution: Let G be a simple graph with p-vertices. Since the graph is simple, therefore G cannot have self loops and parallel edges. Let v be any vertex of G, then v can be adjacent to atmost $(p-1)$ vertices. that is, maximum degree of v is $(p-1)$.

Problem: Show that a simple graph with p vertices can have atmost $\frac{p(p-1)}{2}$ edges.

Solution: We know that,

number of edges in a graph, $q = \frac{\sum \deg(v_i)}{2}$

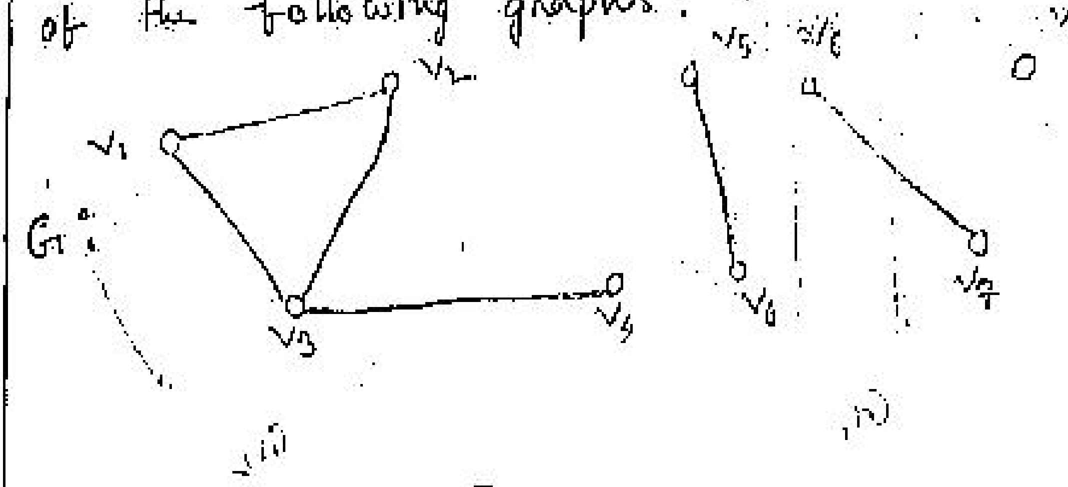
Again in the simple graph, maximum degree of any vertex is $(p-1)$

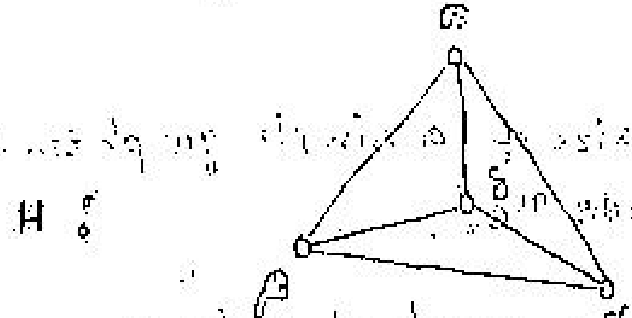
maximum number of edges

$$= \frac{\sum (\text{maximum } \deg(v_i))}{2}$$

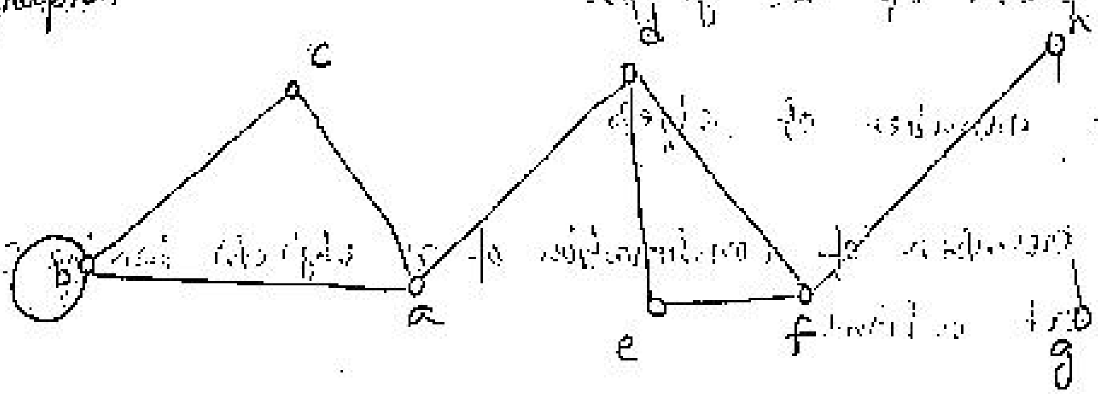
$$= \frac{p(p-1)}{2}$$

problem: Write down the vertex set and edge set of the following graphs:





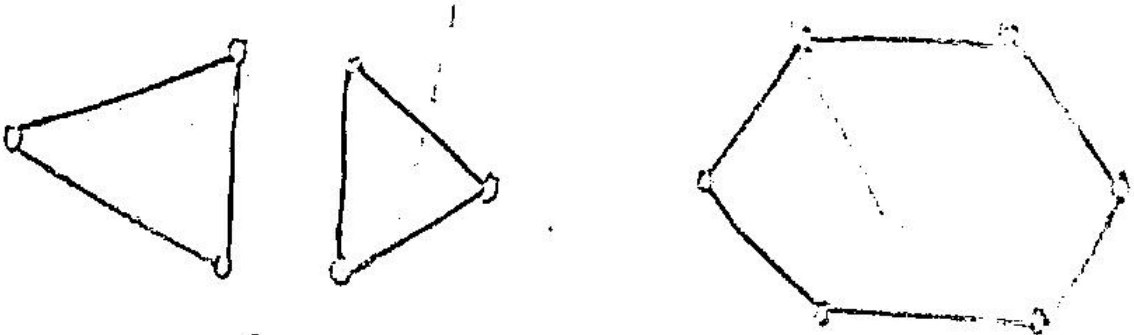
problem:- Find the degree sequence of the following graph:



problem: Construct two graphs having same degree sequence.

Soln: Let the degree sequence of the graphs is
2, 2, 2, 2, 2, 2,

∴ Two graphs having the degree sequence is
2, 2, 2, 2, 2, 2 are,



G_1

G_2

<u>problem:</u> Show that the size of a simple graph of order n cannot exceeds $^nC_2$.

<u>Soln</u>: Let $G = (V, E)$ be a graph of order n. Then number of vertices in G is n. Since, each ~~vertex~~ edge incident with two vertices

∴ Size of the graph

= number of edges

= number of combination of n objects taking 2 at a time

$= {}^nC_2$