we can neglected terms of second and higher orders in $|\psi_{\mu\nu}|$.

We obtain then supposing the field as static (i.e. $\frac{\partial g_{\mu\nu}}{\partial x^4} = 0$

$$\Gamma^{\mu}_{44} = g^{\mu\alpha}\Gamma_{44,\alpha} = g^{\mu\mu}\Gamma_{44,\mu}$$

$$= g^{\mu\mu}\frac{1}{2}\left(\frac{\partial g_{4\mu}}{\partial x^4} + \frac{\partial g_{4\mu}}{\partial x^4} - \frac{\partial g_{44}}{\partial x^{\mu}}\right)$$

$$= \frac{1}{2g_{\mu\mu}}\left(-\frac{\partial g_{44}}{\partial x^{\mu}}\right)$$

$$= \frac{1}{2(-1+\psi_{\mu\mu})}\left\{-\frac{\partial(1+\psi_{44})}{\partial x^{\mu}}\right\}$$

$$\left[\begin{aligned} &\eta_{11} = \eta_{22} = \eta_{33} = -1 \\ &\eta_{44} = 1 \\ &g_{\mu\mu} = \eta_{\mu\mu} + \psi_{\mu\mu} \\ &\quad = -1 + \psi_{\mu\mu} \\ &g_{44} = \eta_{44} + \psi_{44} \\ &\quad = 1 + \psi_{44} \end{aligned}\right.$$

$$\therefore \Gamma^{\mu}_{44} = \frac{1}{2(-1+\psi_{\mu\mu})}\left\{-\frac{\partial \psi_{44}}{\partial x^{\mu}}\right\}, \quad \mu = 1,2,3$$

$$= -\frac{1}{2}(1-\psi_{\mu\mu})^{-1}\left\{-\frac{\partial \psi_{44}}{\partial x^{\mu}}\right\}$$

$$= \frac{1}{2}(1+\psi_{\mu\mu})\frac{\partial \psi_{44}}{\partial x^{\mu}}$$

$$= \frac{1}{2}\frac{\partial \psi_{44}}{\partial x^{\mu}} \longrightarrow (6)$$

In Galilian coordinates,

$$x^1 = x, \quad x^2 = y, \quad x^3 = z, \quad x^4 = ct.$$

$$\therefore ds^2 = -dx^2 - dy^2 - dz^2 + c^2dt^2$$

$$= -v^2dt^2 + c^2dt^2$$

$$= c^2\left(1 - \frac{v^2}{c^2}\right)dt^2 \longrightarrow (7)$$

For small velocities, $\frac{v}{c} << 1$

$$\therefore \quad ds \simeq c\,dt = dx^4 \longrightarrow (8)$$

Hence in weak static field (i.e. it does not change with time), the velocity components can be taken as

$$\frac{dx^1}{ds} = \frac{dx^2}{ds} = \frac{dx^3}{ds} = 0 \; ; \quad \frac{dx^4}{ds} = 1 .$$

By virtue of above, equations (2) becomes

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{44} \frac{dx^4}{ds}\frac{dx^4}{ds} = 0 , \quad \mu = 1, 2, 3 .$$

$$\Rightarrow \frac{d^2x^\mu}{ds^2} = -\Gamma^\mu_{44}$$

Using equation (6), we get

$$\frac{d^2x^\mu}{ds^2} = -\frac{1}{2}\frac{\partial \Psi_{44}}{\partial x^\mu}$$

$$\Rightarrow \frac{d^2x^\mu}{dt^2} = -\frac{c^2}{2}\frac{\partial \Psi_{44}}{\partial x^\mu} \; ; \; \mu = 1, 2, 3 \longrightarrow (9)$$

$$(\because \; ds = c\,dt)$$

Now Newton's equation of motion

$$\frac{d^2x^\mu}{dt^2} = -\frac{\partial \phi}{\partial x^\mu} \; ; \; \text{~~where~~} \longrightarrow (10)$$

where ϕ is the potential function.

Equations (9) and (10) become identical if

$$-\frac{\partial}{\partial x^\mu}\left(\frac{c^2}{2}\Psi_{44}\right) = -\frac{\partial \phi}{\partial x^\mu}$$

Integrating we get,

$$\int \frac{\partial \Psi_{44}}{\partial x^\mu} dx^\mu = \frac{2}{c^2}\int \frac{\partial \phi}{\partial x^\mu} dx^\mu + \text{constant}.$$

$$\Rightarrow \Psi_{44} = \frac{2\phi}{c^2} + K_1$$

Scanned with CamScanner

$$\Rightarrow 1 + \Psi_{44} = \frac{2\phi}{c^2} + K \qquad \text{where } K = 1 + K_1$$

$$\Rightarrow g_{44} = \frac{2\phi}{c^2} + K$$

Since in flat space-time, $g_{44} = 1$, $\phi = 0$

$$\therefore \quad K = 1$$

$$\therefore \quad \boxed{g_{44} = 1 + \frac{2\phi}{c^2}}$$

Hence geodesic equations are reducible to Newton's equation of motion in case of weak static field if $g_{44} = 1 + \frac{2\phi}{c^2}$.

Thus Newton's Theory of gravitation can be regarded as giving the first approximation to the general theory with the quantity g_{44} of General Theory closely related to the Gravitational potential of the Newton's theory.

~·~·~