$$= -\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}\right].$$

[using (7)]

$$= -\rho\frac{du}{dt} \longrightarrow (8.a)$$

where $\frac{d}{dt} = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} + \frac{\partial}{\partial t}$

similarly for $\mu = 2, 3$; equation (6) gives

$$\frac{\partial P_{yx}}{\partial x} + \frac{\partial P_{yy}}{\partial y} + \frac{\partial P_{yz}}{\partial z} = -\rho\frac{dv}{dt} \longrightarrow (8.b)$$

and $$\frac{\partial P_{zx}}{\partial x} + \frac{\partial P_{zy}}{\partial y} + \frac{\partial P_{zz}}{\partial z} = -\rho\frac{\partial w}{\partial t} \longrightarrow (8.c)$$

Here $\frac{du}{dt}, \frac{dv}{dt}, \frac{dw}{dt}$ represent the components of acceleration of the element of the fluid.

Equation (8.a), (8.b) and (8.c) are well-known equations of motion in hydrodynamics, in the absence of any external forces.

Equations (7) and (8) express directly the conservation of mass and momentum so that in Galilian coordinates the principle of conservation of mass and momentum contained in equation (6). i.e. $\frac{\partial T^{\mu\nu}}{\partial x^{\nu}} = 0$. As the Christoffel symbols $\Gamma^{\alpha}_{\mu\nu} = 0$ relative to ~~Galili~~ Galilian coordinates: $T^{\mu\nu}_{\ \ ,\nu} = 0$

Therefore in Galilian coordinates equation (6) may be expressed as $T^{\mu\nu}_{,\nu} = 0$. This equation represents the divergence of $T^{\mu\nu}$ is zero.

(14) To ~~decil~~ derive the formula for energy momentum tensor for a perfect fluid in the form

$$T^{\mu}_{\nu} = (\rho + p) v^{\mu} v_{\nu} - g^{\mu}_{\nu} p .$$

→ In case of a perfect fluid which we define as a mechanical medium incapable of exciting transverse stresses, the only components of stress for a local observer will be those corresponding to the proper hydrostatic pressure p so that energy momentum tensor will in proper coordinates the simple set of components

$T_0^{11} = T_0^{22} = T_0^{33} = p_0$, $T_0^{44} = \rho_0$

where p_0 and ρ_0 denote respectively the pressure and density of a perfect fluid in proper coordinate system.

In proper coordinate system, Galilian coordinate system holds for which

$$ds^2 = -dx^2 - dy^2 - dz^2 + dt^2$$

~~where c=1.~~

where $c = 1$, i.e. the motion of the fluid is considered in gravitational system.

Let g_0^{ij} denote the fundamental tensors in Galilian coordinate system so that

$$g_0^{11} = g_0^{22} = g_0^{33} = -1 \; ; \quad g_0^{44} = 1 \; , \quad g^{ij} = 0 \; (i \neq j)$$

Let T^{ij} and g^{ij} respectively denote energy tensor and fundamental tensor in arbitrary coordinate system.

By tensor law of transformation,

$$T^{ij} = T_0^{ab} \frac{\partial x^i}{\partial x_0^a} \frac{\partial x^j}{\partial x_0^b}$$

$$= \sum_{a=1}^{4} T_0^{aa} \frac{\partial x^i}{\partial x_0^a} \frac{\partial x^j}{\partial x_0^a} \quad \text{according to (1)}$$

$$\Rightarrow T^{ij} = P_0 \sum_{a=1}^{3} \frac{\partial x^i}{\partial x_0^a} \frac{\partial x^j}{\partial x_0^a} + \rho_0 \frac{\partial x^i}{\partial x_0^4} \frac{\partial x^j}{\partial x_0^4} \longrightarrow (2)$$

$$g^{ij} = g_0^{ab} \frac{\partial x^i}{\partial x_0^a} \frac{\partial x^j}{\partial x_0^b}$$

$$= \sum_{a=1}^{4} g_0^{aa} \frac{\partial x^i}{\partial x_0^a} \frac{\partial x^j}{\partial x_0^a}$$

$$= - \sum_{a=1}^{3} \frac{\partial x^i}{\partial x_0^a} \frac{\partial x^j}{\partial x_0^a} + \frac{\partial x^i}{\partial x_0^4} \frac{\partial x^j}{\partial x_0^4}$$

$$\Rightarrow \sum_{a=1}^{3} \frac{\partial x^i}{\partial x_0^a} \frac{\partial x^j}{\partial x_0^a} = -g^{ij} + \frac{\partial x^i}{\partial x_0^4} \frac{\partial x^j}{\partial x_0^4}$$

Using this result in (2), we get

$$T^{ij} = P_0 \left[-g^{ij} + \frac{\partial x^i}{\partial x_0^4} \frac{\partial x^j}{\partial x_0^4} \right] + \rho_0 \frac{\partial x^i}{\partial x_0^4} \frac{\partial x^j}{\partial x_0^4}$$

$$\Rightarrow T^{ij} = (P_0 + \rho_0) \frac{\partial x^i}{\partial x_0^4} \frac{\partial x^j}{\partial x_0^4} - P_0 g^{ij} \longrightarrow (3)$$

Since the fluid is at rest in the proper coordinate system and hence the velocity components can be written as

$$\frac{dx_0^1}{ds} = \frac{dx_0^2}{ds} = \frac{dx_0^3}{ds} = 0, \quad \frac{dx_0^4}{ds} = 1 \longrightarrow (4)$$

$$\therefore \frac{dx^i}{ds} = \frac{\partial x^i}{\partial x_0^j} \frac{dx_0^j}{ds} = \frac{\partial x^i}{\partial x_0^4} \frac{dx_0^4}{ds}$$

$$\Rightarrow \frac{dx^i}{ds} = \frac{\partial x^i}{\partial x_0^4}$$

Using this result in (3), we have

$$T^{ij} = (P_0 + \rho_0) \frac{dx^i}{ds} \frac{dx^j}{ds} - P_0 g^{ij}$$

Taking $P_0 = P$ and $\rho_0 = \rho$ we get

$$T^{ij} = (P+\rho) \frac{dx^i}{ds} \frac{dx^j}{ds} - P g^{ij}$$

$$= (P+\rho) v^i v^j - P g^{ij},$$

where $v^i = \frac{dx^i}{ds}$

= velocity components

This can be also expressed as

$$T^{\mu}_{\nu} = (P+\rho) v^{\mu} v_{\nu} - P g^{\mu}_{\nu}$$

$$T_{\mu\nu} = (P+\rho) v_{\mu} v_{\nu} - P g_{\mu\nu}$$

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