The most appropriate tensor which contains second order derivatives is the Ricci-tensor $R_{\mu\nu}$. Hence L.H.S of (1) will be either $R_{\mu\nu}$ or its linear combination while describing the relativistic field equation. We must keep in mind that the field equation must be invariant under tensor law of transformation. It means that both the sides of (1) must be expressed in terms of tensor. Hence ρ in (1) must be replaced be second rank tensor. This tensor is the energy momentum tensor.

Hence Einstein to proposed to proposed the following field equations in general relativity.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -k T_{\mu\nu} \longrightarrow (5)$$

where k stands for constant $= 8\pi$.
If we consider gravitation in empty space due to some source, then the most general form of field equation (5), (taking cosmological constant Λ into account) is written as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -8\pi T_{\mu\nu} \longrightarrow (6)$$

where Λ (by Einstein) is the cosmological constant and is a very small quantity.

Equations (6) are the required field equations in the general theory of relativity in presence of matter and represented the Einstein law of gravitation for naturally curved material world.

Multiplying equation (6) throughout by $g^{\mu\nu}$, we get

$$g^{\mu\nu} R_{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + \Lambda g^{\mu\nu} g_{\mu\nu} = -8\pi g^{\mu\nu} T_{\mu\nu}$$

$$\Rightarrow R - \frac{1}{2} \cdot 4 \cdot R + \Lambda \cdot 4 = -8\pi T$$

$$\Rightarrow -R + 4\Lambda = -8\pi T$$

$$\Rightarrow R - 4\Lambda = 8\pi T$$

In the absence of matter,

$$T_{\mu\nu} = 0 \Rightarrow T = 0$$

$$\therefore \quad R = 4\Lambda$$

Using this results in (6) we have

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} 4\Lambda + \Lambda g_{\mu\nu} = -8\pi \cdot 0$$

$$\Rightarrow R_{\mu\nu} - \Lambda g_{\mu\nu} = 0$$

$$\Rightarrow R_{\mu\nu} = \Lambda g_{\mu\nu}$$

These equations represent Einstein field equations in the general theory of relativity in absence of matter or Einstein's law of gravitation in empty space.

of $\Lambda = 0$, the Einstein's law of graitation in empty space is

$$R_{\mu\nu} = 0 \longrightarrow (7)$$

Thus the field equation in empty space are given by (7). These equations are $\frac{4(4+1)}{2} = 10$ in number. All of them are