Self - similar fractals: A fractal is a geometric object that is similar to itself on all scales. If we zoom in on a fractal object it will look similar or exactly like the original shape. This property is called self similarity. That is self similarity mean an object that made of small copies of itself. Example : Von Koch curve, Cantor middle third set etc.

Measure and mass distributions:

We call 'μ' a measure on $\mathbb{R}^n$ [n-dimensional Euclidean space] if μ assigns a non negative ~~integer~~ number, possibly ∞, to each subset of $\mathbb{R}^n$ such that :

(a) $\mu(\phi) = 0$

(b) $\mu(A) \leq \mu(B)$ if $A \subseteq B$

(c) If $A_1, A_2, \ldots$ is countable (or finite) sequence of sets then

$$\mu\left(\bigcup_{i=1}^{\infty} A_i\right) \leq \sum_{i=1}^{\infty} \mu(A_i)$$

We call $\mu(A)$ the measure of the set A, and think of $\mu(A)$ as the size of A measured in some way. Condition (a) says that the empty set has measure zero, Condition (b) says that the larger the set, the larger the measure and (c) says that if a set is a union of a countable number of pieces then the sum of measure of the pieces is at least equal to the measure of the whole.

A measure on a bounded subset of $\mathbb{R}^n$ for which $0 < \mu(\mathbb{R}^n) < \infty$ will be called a mass distribution, and we think $\mu(A)$ as the mass of the set A.

<u>Example:</u> For each subset A of $\mathbb{R}^n$ let $\mu(A)$ be the number of points in A if A is finite and ∞ otherwise. Then μ is a measure on $\mathbb{R}^n$.

Let a be a point in $\mathbb{R}^n$ and define $\mu(A)$ to be 1 if A contains A and 0 otherwise. Then μ is a mass distribution.

* Hausdorff measure and dimension:

* Hausdorff measure:

If U is any non empty subset of n-dimensional Euclidean space $\mathbb{R}^n$, the diameter of U is defined as $|U| = \sup\{|x-y| : x, y \in U\}$, i.e the greatest distance apart of any pair of points in U. If $\{U_i\}$ is a countable (or finite) collection of sets of diameter at most δ that cover F, i.e $F \subset \bigcup_{i=1}^{\infty} U_i$ with $0 < |U_i| \le \delta$ for each i, we say that $\{U_i\}$ is a δ-cover of F.

Suppose that F is a subset of $\mathbb{R}^n$ and 's' is a non negative number. For any $\delta > 0$ we define

$$\mathcal{H}^s_\delta(F) = \inf\left\{ \sum_{i=1}^{\infty} |U_i|^s : \{U_i\} \text{ is a } \delta\text{-cover of } F \right\}$$

we look at all covers of F by sets of diameter at most δ and seek to minimize the sum of the sth powers of the diameters. As δ decreases, the class of permissible covers of F is reduced. Therefore, the infimum $\mathcal{H}^s_\delta(F)$ increases, and so approaches a limit as $\delta \to 0$.

We write

$$\mathcal{H}^s(F) = \lim_{\delta \to 0} \mathcal{H}^s_\delta(F).$$

This limit exist for any subset F of $\mathbb{R}^n$, though the limiting value can be 0 or ∞. We call $\mathcal{H}^s(F)$ the ~~s~~ s- dimensional Hausdorff measure of F.