not independent to each other. Some of them satisfy the four Bianchi identities. As a resuld of which these are reduced to 6 in number. We are to determine 10 unknown components $g_{\mu\nu}$ from these equations. Hence we can not determine them uniquely. Therefore in any gravitatinal situation all the 10 $g_{\mu\nu}$ can not be uniquely determined.

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⊛ Poisson's Equation as an approximation of Einstein's field Equations:

Assuming cosmological constants Λ to be very small quantity, Einstein's field equations are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi T_{\mu\nu} \longrightarrow (1)$$

In order to obtain Newtonian approximation of above equations, consider the motion of the test particle in a very weak static field which is characterized by

$$g_{\mu\nu} = \epsilon_{\mu\nu} + \eta_{\mu\nu} \longrightarrow (2)$$

where $\epsilon_{\mu\nu}$ is the matric tensor for Euclidean space and $\eta_{\mu\nu}$ is the function of (x, y, z) and is so small that the powers of $\eta_{\mu\nu}$ higher than the first are neglected. In this case (for weak static field) the line element will differ very slightly from that of the special relativity and we must have

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$$\left.\begin{aligned}\epsilon_{11} = \epsilon_{22} = \epsilon_{33} = -\epsilon_{44} = -1 \quad & \text{and} \\ \epsilon_{\mu\nu} = g_{\mu\nu} = 0 \quad \text{for } \mu \neq \nu & \end{aligned}\right\} \longrightarrow (3)$$

Since the fluid is static, i.e. it does not change with time and hence velocity components can be taken as

$$\frac{dx^1}{ds} = \frac{dx^2}{ds} = \frac{dx^3}{ds} = 0, \quad \frac{dx^4}{ds} = 1 \longrightarrow (4)$$

The coordinates considered are Galilean coordinates,

$$x^1 = x, \quad x^2 = y, \quad x^3 = z, \quad x^4 = ct.$$

The geodesic equations are reduced to Newtonian equations of motion if

$$g_{44} = \frac{2\psi}{c^2} + 1 = 1 + 2\psi, \quad \text{taking } c = 1$$

All the components of energy tensor (in the limit of Newtonian approximation) will be approximately equal to zero except

$$T^{44} = \rho \frac{dx^4}{ds}\frac{dx^4}{ds} = \rho$$

So that

$$T = g^{\mu\nu} T_{\mu\nu} = g^{44} T_{44} = \frac{1}{g_{44}} T_{44}$$

$$\Rightarrow T = \frac{1}{1 + \eta_{44}} T_{44} = (1 + \eta_{44})^{-1} T_{44}$$

$$\Rightarrow T = (1 - \eta_{44} + \cdots\cdots) T_{44} \quad ; \quad T_{44} = \rho$$

$$\therefore T_{44} = \rho = T$$

From the field equation (1) we have

$$g^{\mu\nu} R_{\mu\nu} - \frac{1}{2} g^{\mu\nu} g_{\mu\nu} R = -8\pi g^{\mu\nu} T_{\mu\nu}$$

$$\Rightarrow R - \frac{4}{2} R = -8\pi T$$

$$\Rightarrow R = 8\pi T.$$