$$\therefore \quad R_{\mu\nu} = -8\pi\, T_{\mu\nu} + \frac{1}{2} g_{\mu\nu} T \cdot 8\pi$$

$$= -8\pi \left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}\right)$$

$$\therefore \quad R_{44} = -8\pi \left(T_{44} - \frac{1}{2} T g_{44}\right) = -8\pi\rho\left(1 - \frac{1}{2} g_{44}\right)$$

$$= -8\pi\rho\left(1 - \frac{1}{2}\times 1\right)$$

$$\Rightarrow \quad R_{44} = -4\pi\rho \longrightarrow (5)$$

Since $R_{\mu\nu} = \frac{\partial}{\partial x^{\nu}} \Gamma^{a}_{\mu a} - \frac{\partial}{\partial x^{a}} \Gamma^{a}_{\mu\nu} + \Gamma^{\lambda}_{\mu a} \Gamma^{a}_{\lambda\nu} - \Gamma^{\lambda}_{\mu\nu} \Gamma^{a}_{\lambda a}$

$$\therefore \quad R_{44} = \frac{\partial}{\partial x^{4}} \Gamma^{a}_{4a} - \frac{\partial}{\partial x^{a}} \Gamma^{a}_{44} + \Gamma^{\lambda}_{4a} \Gamma^{a}_{\lambda 4} - \Gamma^{\lambda}_{44} \Gamma^{a}_{\lambda a}$$

Writing upto first order of approximation,

$$R_{44} = \frac{\partial}{\partial x^{4}} \Gamma^{a}_{4a} - \frac{\partial}{\partial x^{a}} \Gamma^{a}_{44}$$

Since $g_{\mu\nu}$ are not the function of time in static gravitational field.

$$\therefore \quad R_{44} = -\frac{\partial}{\partial x^{a}} \Gamma^{a}_{44} \longrightarrow (6) \quad ; \quad a = 1, 2, 3$$

combining (5) and (6) we get

$$-4\pi\rho = -\frac{\partial \Gamma^{a}_{44}}{\partial x^{a}}$$

$$\Rightarrow \frac{\partial \Gamma^{a}_{44}}{\partial x^{a}} = 4\pi\rho \longrightarrow (7)$$

Also for weak static field for $a = 1, 2, 3$; we have

$$\Gamma^{a}_{44} = g^{ab}\, \Gamma_{44,b} = g^{aa}\, \Gamma_{44,a}$$

$$\Rightarrow \Gamma^{a}_{44} = \frac{1}{g_{aa}} \Gamma_{44,a}$$

$$\Rightarrow \Gamma^a_{44} = \frac{1}{-1+\eta_{aa}}\left(-\frac{1}{2}\frac{\partial g_{44}}{\partial x^a}\right)$$

$$= (1-\eta_{aa})^{-1}\left(\frac{1}{2}\frac{\partial g_{44}}{\partial x^a}\right)$$

$$= \frac{1}{2}\frac{\partial g_{44}}{\partial x^a}$$

Now (7) can be expressed as

$$\frac{\partial}{\partial x^a}\left[\frac{1}{2}\frac{\partial g_{44}}{\partial x^a}\right] = 4\pi\rho$$

$$\Rightarrow \sum_{a=1}^{3}\frac{\partial^2 g_{44}}{(\partial x^a)^2} = 8\pi\rho$$

$$\Rightarrow \frac{\partial^2 g_{44}}{\partial x^2} + \frac{\partial^2 g_{44}}{\partial y^2} + \frac{\partial^2 g_{44}}{\partial z^2} = 8\pi\rho$$

$$\Rightarrow \nabla^2 g_{44} = 8\pi\rho$$

$$\cancel{\Rightarrow \nabla^2\psi = 4\pi\rho}$$

$$\Rightarrow \nabla^2(1+2\psi) = 8\pi\rho$$

$$\Rightarrow \nabla^2\psi = 4\pi\rho$$

which is the ~~equation~~ poission's equations in relativistic units $c=1$, $G=1$. Thus the relativistic general theory of gravitations corresponds to the Newtonian theory of gravitation in the presence of matter in the non-relativistic limit of a weak, ~~so~~ static gravitational field.

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