⊛ Solution of the Einstein's gravitational equations in empty space:

or Schwarzschild exterior solution for a gravitating field of an isolated particle:

→ The first exact solution of the Einstein equations was obtained by Schwarzschild (1916) for a static and spherically symmetric field, which is a good approximation for the gravitational field of our sun.

In empty space the law of gravitation chosen by Einstein is

$$R_{\mu\nu} = 0 \longrightarrow (1)$$

Einstein modified equation (1) later on and cosmological constant Λ was included and took

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \longrightarrow (2)$$

as field equations in empty space.

The solution of the above equations consists of finding the line element for interval in empty space surrounding a gravitating point particle, which ultimately corresponds to the field of an isolated particle continually at rest at the origin. This solution was first given by Schwarzschild and is of great importance since it provides a treatment of the gravitational field. Surrounding the sun for use a discussing three crucial tests, that distinguish between the predictions of the Newtonian theory of gravitation and the more exact predictions of the theory of relativity.

In absence of any mass (empty space) the space-time would be flat so that the line element in spherical polar coordinates would be expressed as

$$ds^2 = -dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2 + dt^2 \longrightarrow (3)$$

The presence of the gravitating mass point modify the space-time. However, since the mass is static and isolated, the line-element would be spatially spherically symmetric about the point mass and is static. The most general form of such a line element may be expressed as

$$ds^2 = -e^{2\lambda} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2 + e^{2\nu} dt^2$$

$$= g_{\mu\nu} dx^\mu dx^\nu \longrightarrow (4)$$

with $x^1 = r$, $x^2 = \theta$, $x^3 = \phi$, $x^4 = t$ and where λ and ν are functions of r only, since for spherically symmetric isolated particle the field will depend on r alone, not on θ and ϕ.

Since the gravitational field, i.e. disturbance from flat space-time due to a particle diminishes as we go to an infinite distance. Hence the line-element (4) reduce to Galilean line-element (3) at an infinite distance from the particle.