Ex:- If $e = \sin\psi$, prove that the relation between the true anomaly ν and eccentric anomaly E is

2017

$$\tan\frac{\nu}{2} = \tan\left(45^\circ + \frac{\psi}{2}\right)\tan(E/2).$$

Sol^n We have ~~the~~ $\tan\frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$.

$$= \sqrt{\frac{1+\sin\psi}{1-\sin\psi}}\tan\frac{E}{2}$$

$[\because e = \sin\psi]$

$$= \sqrt{\frac{(\cos\psi/2 + \sin\frac{\psi}{2})^2}{(\cos\frac{\psi}{2} - \sin\frac{\psi}{2})^2}}\tan\frac{E}{2}$$

$$= \frac{\cos\frac{\psi}{2} + \sin\frac{\psi}{2}}{\cos\frac{\psi}{2} - \sin\frac{\psi}{2}} \tan\frac{E}{2}$$

$$= \frac{1 + \tan\frac{\psi}{2}}{1 - \tan\frac{\psi}{2}} \tan\frac{E}{2}$$

$$= \tan\left(45^\circ + \frac{\psi}{2}\right)\tan\frac{E}{2}$$

Ex Express ~~the~~ true anomaly ν in terms of e and eccentric anomaly E

2018 Soln we have $\tan\frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$. $\rightarrow$ ①

Let us define an angle ψ lying between 0 and $\pi/2$ by $\sin\psi = e$ $\rightarrow$ ②

$$\tan\frac{\nu}{2} = \sqrt{\frac{1+\sin\psi}{1-\sin\psi}} \tan\frac{E}{2}$$

$$= \frac{\cos\frac{\psi}{2} + \sin\frac{\psi}{2}}{\cos\frac{\psi}{2} - \sin\frac{\psi}{2}} \tan\frac{E}{2}.$$

Let $\tan\frac{\psi}{2} = x$, then we get

$$\tan\frac{\nu}{2} = \frac{1+x}{1-x}\tan\frac{E}{2}$$

$$\Rightarrow \frac{2i\sin\frac{\nu}{2}}{2\cos\frac{\nu}{2}} = \frac{1+x}{1-x}\frac{2i\sin\frac{E}{2}}{2\cos\frac{E}{2}}$$

$$\Rightarrow \frac{e^{i\nu/2}-e^{-i\nu/2}}{e^{i\nu/2}+e^{-i\nu/2}} = \frac{1+x}{1-x}\cdot\frac{e^{iE/2}-e^{-i\frac{E}{2}}}{e^{iE/2}+e^{-iE/2}}$$

$$\left[\because 2\cos\theta = e^{i\theta}+e^{-i\theta}\right.$$
$$\left. 2i\sin\theta = e^{i\theta}-e^{-i\theta}\right]$$

By components and dividendo, we get

$$\frac{2e^{i\nu/2}}{-2e^{-i\nu/2}} = \frac{(1+x)\left(e^{iE/2}-e^{-iE/2}\right)+(1-x)\left(e^{iE/2}+e^{-iE/2}\right)}{(1+x)\left(e^{iE/2}-e^{-iE/2}\right)-(1-x)\left(e^{iE/2}+e^{-iE/2}\right)}$$

$$\Rightarrow -e^{i\nu} = \frac{2\left(e^{iE/2}-xe^{-iE/2}\right)}{2\left(xe^{iE/2}-e^{-iE/2}\right)}$$

$$\Rightarrow e^{i\nu} = \frac{e^{iE/2}\left(1-xe^{-iE}\right)}{e^{-iE/2}\left(1-xe^{iE}\right)}$$

$$= \frac{e^{iE}(1-xe^{-iE})}{(1-xe^{iE})}$$

taking logarithms, we get

$$iv = iE + \log(1-xe^{-iE}) - \log(1-xe^{iE})$$

$$= iE + \left[-xe^{-iE} - \frac{1}{2}x^2e^{-2iE} - \frac{1}{3}x^3e^{-3iE} \cdots\cdots\right]$$

$$- \left[-xe^{iE} - \frac{1}{2}x^2e^{2iE} - \frac{1}{3}x^3e^{3iE} \cdots\cdots\right]$$

$$= iE + x\,2i\sin E + \frac{1}{2}x^2\,2i\sin 2E + \frac{1}{3}x^3\,2i\sin 3E + \cdots$$

Equating imaginary parts, we get

$$v = E + 2x\sin E + \frac{1}{2}x^2 \cdot 2\sin 2E + \frac{1}{3}x^3\,2\sin 3E + \cdots\cdots$$

Now, $x = \tan\frac{\psi}{2} = \frac{\sin\frac{\psi}{2}}{\cos\frac{\psi}{2}} = \frac{2\sin^2\frac{\psi}{2}}{2\sin\frac{\psi}{2}\cos\frac{\psi}{2}} = \frac{1-\cos\psi}{\sin\psi}$

$$= \frac{1-\sqrt{1-e^2}}{e}$$

$$= \frac{1-(1-\frac{1}{2}e^2 - \frac{1}{8}e^4 \cdots\cdots)}{e}$$

$$\therefore \partial = E + \left(e + \frac{1}{4}e^3\right)\sin E + \frac{1}{4}e^2 \sin 2E + \frac{1}{12}e^3 \sin 3E$$

(Correct the 3rd degree of e //