$\because \lambda \to 0,\ \nu \to 0$ as $r \to \infty$

$$\therefore g_{\mu\nu} = \begin{bmatrix} -e^{2\lambda} & 0 & 0 & 0 \\ 0 & -r^2 & 0 & 0 \\ 0 & 0 & -r^2\sin^2\theta & 0 \\ 0 & 0 & 0 & e^{2\nu} \end{bmatrix} \longrightarrow (5)$$

$$\therefore g = |g_{\mu\nu}| = -r^4 e^{2(\lambda+\nu)} \sin^2\theta \longrightarrow (6)$$

The element of the matrix corresponding to the inverse matrix tensor are

$$g^{\mu\nu} = \frac{\text{cofactor of } g_{\mu\nu}}{g} \longrightarrow (7)$$

$$g = -e^{2\lambda+2\nu} r^4 \sin^2\theta$$

$$\therefore \sqrt{-g} = e^{\lambda+\nu} r^2 \sin\theta$$

$$\Rightarrow \log\sqrt{-g} = (\lambda+\nu)\log e + \log r^2 + \log\sin\theta$$

$$= (\lambda+\nu) + 2\log r + \log\sin\theta.$$

$$\therefore \frac{\partial}{\partial r}\log\sqrt{-g} = (\lambda'+\nu') + \frac{2}{r} + 0$$

$$\frac{\partial^2}{\partial r^2}\log\sqrt{-g} = (\lambda''+\nu'') - \frac{2}{r^2}$$

$$\frac{\partial}{\partial\theta}\log\sqrt{-g} = \cot\theta \ ; \quad \frac{\partial}{\partial\phi}\log\sqrt{-g} = 0.$$

$$\frac{\partial^2}{\partial\theta^2}\log\sqrt{-g} = -\operatorname{cosec}^2\theta$$

The surviving christoffel's symbol calculated by using the formula viz,

$$\Gamma_{\mu\nu,a} = \frac{1}{2}\left(\frac{\partial g_{\nu a}}{\partial x^{\mu}} + \frac{\partial g_{\mu a}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{a}}\right)$$

and $\Gamma^{\alpha}_{\mu\nu} = g^{\alpha\beta}\Gamma_{\mu\nu,\beta}$ are

$$\Gamma_{12}^{2} = g^{22}\,\Gamma_{12,2} = g^{22}\,\frac{1}{2}\,\frac{\partial g_{22}}{\partial x^1}$$

$$= \frac{1}{2}\left(-\frac{1}{r^2}\right)\frac{\partial}{\partial r}\left(-r^2\right) = \frac{1}{r}$$

$$g_{11} = -e^{2\lambda}, \quad g_{22} = -r^2, \quad g_{33} = -r^2\sin^2\theta, \quad g_{44} = e^{2\nu}$$

$$\Gamma_{13}^{3} = \frac{1}{r}, \quad \Gamma_{12}^{1} = g^{11}\,\Gamma_{22,1}$$

$$= -\frac{g^{11}}{2}\,\frac{\partial g_{22}}{\partial x^1} = -\frac{1}{2}\left(-e^{-2\lambda}\right)\left(-2r\right)$$

$$= -r\,e^{-2\lambda}$$

$$\Gamma_{33}^{1} = g^{11}\,\Gamma_{33,1} = g^{11}\,\frac{1}{2}\,\frac{\partial g_{33}}{\partial x^1} = \frac{e^{-2\lambda}}{2}\,\frac{\partial}{\partial r}\left(-r^2\sin^2\theta\right)$$

$$= -r\,e^{-2\lambda}\sin^2\theta.$$

$$\Gamma_{44}^{1} = -g^{11}\,\frac{1}{2}\,\frac{\partial g_{44}}{\partial r} = e^{-2\lambda}\,\frac{1}{2}\,\frac{\partial}{\partial r}\left(e^{2\nu}\right)$$