$$⑤ \Rightarrow -n^2 \vec{r_1} = \frac{k^2}{\rho^3}[m_2(\vec{r_2}-\vec{r_1}) - m_3(\vec{r_1}-\vec{r_3})]$$

$$= \frac{k^2}{\rho^3}[m_2\vec{r_2} + m_3\vec{r_3} - m_2\vec{r_1} - m_3\vec{r_1}]$$

$$= \frac{k^2}{\rho^3}[(m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3}) - (m_1+m_2+m_3)\vec{r_1}]$$

$$= -\frac{k^2}{\rho^3}(m_1+m_2+m_3)\vec{r_1}$$

$$\Rightarrow n^2 = \frac{k^2}{\rho^3}(m_1+m_2+m_3)$$

$$= \frac{k^2}{\rho^3} M, \text{ where } M = m_1+m_2+m_3$$

2018

$\Rightarrow \rho = \left(\frac{k^2 M}{n^2}\right)^{1/3}$, which gives the distance of the masses from the other.

Hence, in this case, the problem is solvable.

Here the distance of each pair of masses are same as they form an equitorial triangle and this solution is called equitatorial triangle solution of the ~~s~~ three body problem.

case ⑥ :- Let us suppose that $\rho_{12} \neq \rho_{31}$.

i.e $\vec{r_1} \times \vec{r_2} = 0$.

2014 (✗)

$\Rightarrow \vec{r_1} \parallel \vec{r_2}$.

Hence, there exist a non-zero scalar λ such that $\vec{r_1} = \lambda \vec{r_2}$.

(10) $\Rightarrow (\vec{r_3} \times \underbrace{\lambda\vec{r_2}}_{\vec{r_1}}) \left(\dfrac{1}{\rho_{31}^3} - \dfrac{1}{\rho_{23}^3}\right) = 0.$

$\Rightarrow (\vec{r_2} \times \vec{r_3}) \left(\dfrac{1}{\rho_{31}^3} - \dfrac{1}{\rho_{23}^3}\right) = 0 \rightarrow (13)$

Now adding (9) & (13) we get

$$(\vec{r_2} \times \vec{r_3}) \left(\frac{1}{\rho_{31}^3} - \frac{1}{\rho_{12}^3}\right) = 0$$

$\Rightarrow \vec{r_2} \times \vec{r_3} = 0.$ $\quad [\because \rho_{12} \neq \rho_{31}]$

$\Rightarrow \vec{r_2}$ is parallel to $\vec{r_3}$

$\therefore \vec{r_1} \parallel \vec{r_2} \parallel \vec{r_3}$

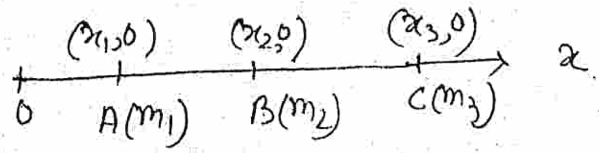
$\therefore$ A, B & C are colinear.

The masses are situated in the same straight line.

Let us assume that three masses are situated on the x-axis, so that

$\vec{r_1} = (x_1, 0)$, $\vec{r_2} = (x_2, 0)$, $\vec{r_3} = (x_3, 0)$

Thus we get



$\rho_{12} = |x_2 - x_1|$, $\rho_{31} = |x_1 - x_3|$

$\rho_{23} = |x_3 - x_2|$

1)

(5) $\Rightarrow$ $-n^2 x_1 = K^2\left[\frac{m_2}{\rho_{12}^3}(x_2-x_1) - \frac{m_3}{\rho_{31}^3}(x_1-x_3)\right] \to (14)$ (15)

(6) $\Rightarrow$ $-n^2 x_2 = K^2\left[\frac{m_3}{\rho_{23}^3}(x_3-x_2) - \frac{m_1}{\rho_{12}^3}(x_2-x_1)\right] \to (15)$ (16)

(7) $\Rightarrow$ $-n^2 x_3 = K^2\left[\frac{m_1}{\rho_{31}^3}(x_1-x_3) - \frac{m_2}{\rho_{31}^3}(x_3-x_2)\right] \to (16)$

Now (15) - (14) $\Rightarrow$ 3)

$$-\frac{n^2}{K^2}(x_2-x_1) = \frac{-(m_1+m_2)(x_2-x_1)}{\rho_{12}^3} + \frac{m_3(x_3-x_2)}{\rho_{23}^3} + \frac{m_3}{\rho_{31}^3}(x_1-x_3)$$

$$\Rightarrow -\frac{n^2}{K^2}\rho_{12} = -\frac{(m_1+m_2)}{\rho_{12}^3}\rho_{12} + \frac{m_3}{\rho_{23}^3}\rho_{23} - \frac{m_3}{\rho_{31}^3}\rho_{31}.$$

$$= -\frac{m_1+m_2}{\rho_{12}^2} + \frac{m_3}{\rho_{23}^2} - \frac{m_3}{\rho_{31}^2}$$

$$= -\frac{m_1+m_2}{\rho_{12}^2} + m_3\left(\frac{1}{\rho_{23}^2} - \frac{1}{\rho_{31}^2}\right) \to (17)$$

8)

Illy,

(16) - (15) $\Rightarrow$

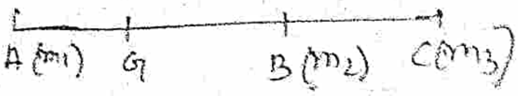
$$-\frac{n^2}{K^2}\rho_{23} = -\frac{m_2+m_3}{\rho_{23}^2} + m_1\left(-\frac{1}{\rho_{31}^2} + \frac{1}{\rho_{12}^2}\right) \to (18)$$

& (14) - (16) $\Rightarrow$

$$\frac{m^2}{K^2}\rho_{31} = \frac{m_3+m_1}{\rho_{31}^2} + m\left(\frac{1}{\rho_{12}^2} + \frac{1}{\rho_{23}^2}\right) \to (19)$$

Let us consider one possible arrangement of the masses as shown in the figure

$$\rho_{13} = \rho_{12} + \rho_{23} \rightarrow (20)$$



Now, (17) $\times \rho_{23}$ − (18) $\times \rho_{12} \Rightarrow$

$$0 = -\frac{(m_1+m_2)}{\rho_{12}^2}\rho_{23} + m_3\left(\frac{1}{\rho_{23}} - \frac{\rho_{23}}{\rho_{31}^2}\right) + \frac{m_2+m_3}{\rho_{23}^2}\rho_{12} - m_1\left(\frac{1}{\rho_{12}} - \frac{\rho_{12}}{\rho_{13}^2}\right)$$

Again multiplying throughout by ρ_{12}, we get

$$-(m_1+m_2)\frac{\rho_{23}}{\rho_{12}} + m_3\left(\frac{\rho_{12}}{\rho_{23}} - \frac{\rho_{23}}{\rho_{13}}\cdot\frac{\rho_{12}}{\rho_{31}}\right) + (m_2+m_3)\left(\frac{\rho_{12}}{\rho_{23}}\right)^2 - m_1\left\{1-\left(\frac{\rho_{12}}{\rho_{13}}\right)^2\right\} = 0 \rightarrow (21)$$

Let us define x as $x = \frac{\rho_{12}}{\rho_{23}}$

since $\rho_{13} = \rho_{12} + \rho_{23}$

$$\Rightarrow \frac{\rho_{13}}{\rho_{23}} = \frac{\rho_{12}}{\rho_{23}} + 1 = x+1.$$

$$\& \quad \frac{\rho_{12}}{\rho_{13}} = \frac{\rho_{12}}{\rho_{23}}\cdot\frac{\rho_{23}}{\rho_{13}} = \frac{x}{1+x}.$$

The eq^n (21) reduces to

$$-(m_1+m_2)\frac{1}{x} + m_3\left(x - \frac{1}{x+1}\cdot\frac{x}{x+1}\right) + (m_2+m_3)x^2 - m_1\left\{1-\frac{x^2}{(x+1)^2}\right\} = 0.$$

Multiplying $x(x+1)^2$, we get.

$$-(m_1+m_2)(x+1)^2+m_3\{x^2(x+1)^2-x^2\}+(m_2+m_3)x^3(x+1)^2$$
$$-m_1\{x(x+1)^2-x^3\}=0$$

$$\Rightarrow -(m_1+m_2)(x^2+2x+1)+m_3\{x^2(x^2+2x+1)-x^2\}+$$
$$(m_2+m_3)x^3(x^2+2x+1)$$
$$-m_1\{x(x^2+2x+1)-x^3\}=0$$

$$\Rightarrow -(m_1+m_2)(x^2+2x+1)+m_3(x^4+2x^3)+(m_2+m_3)(x^5+2x^4+x^3)$$
$$-m_1(2x^2+x)=0$$

$$\Rightarrow (m_2+m_3)x^5+(2m_2+3m_3)x^4+(m_2+3m_3)x^3$$
$$-(3m_1+m_2)x^2-(3m_1+2m_2)x-(m_1+m_2)=0 \longrightarrow (22)$$

Which is called Lagranges eq^n.

eq^n (22) is a fifth degree eq^n & there is only one change in sign in the coefficient of the eq^n. Hence from Dèscartes rule of sign it follows that the eq^n (22) has at least one positive real root x. with this known value of x we can determine

ρ_{13} uniquely from the relation $\rho_{13} = \rho_{23} + \rho_{12}$

2017 with a given value of ρ_{12}.

(10) Hence the problem is solvable. This solution of the three body problem is known as the straight line solⁿ.

Case II:- when the mutual distances between