Alternating Current
Lecture 7

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Average value of alternating current:

The average value of alternating current is defined as its average taken over half of the period. Hence we can write the following equation as

$$i_{av} = \frac{1}{\pi}\int_0^{\pi} idt \rightarrow (i)$$

$$\Rightarrow i_{av} = \frac{1}{\pi}\int_0^{\pi} i_m sin\omega t dt$$

$$\Rightarrow I_{av} = \frac{i_m}{\pi}[-cos\omega t]_0^{\pi}$$

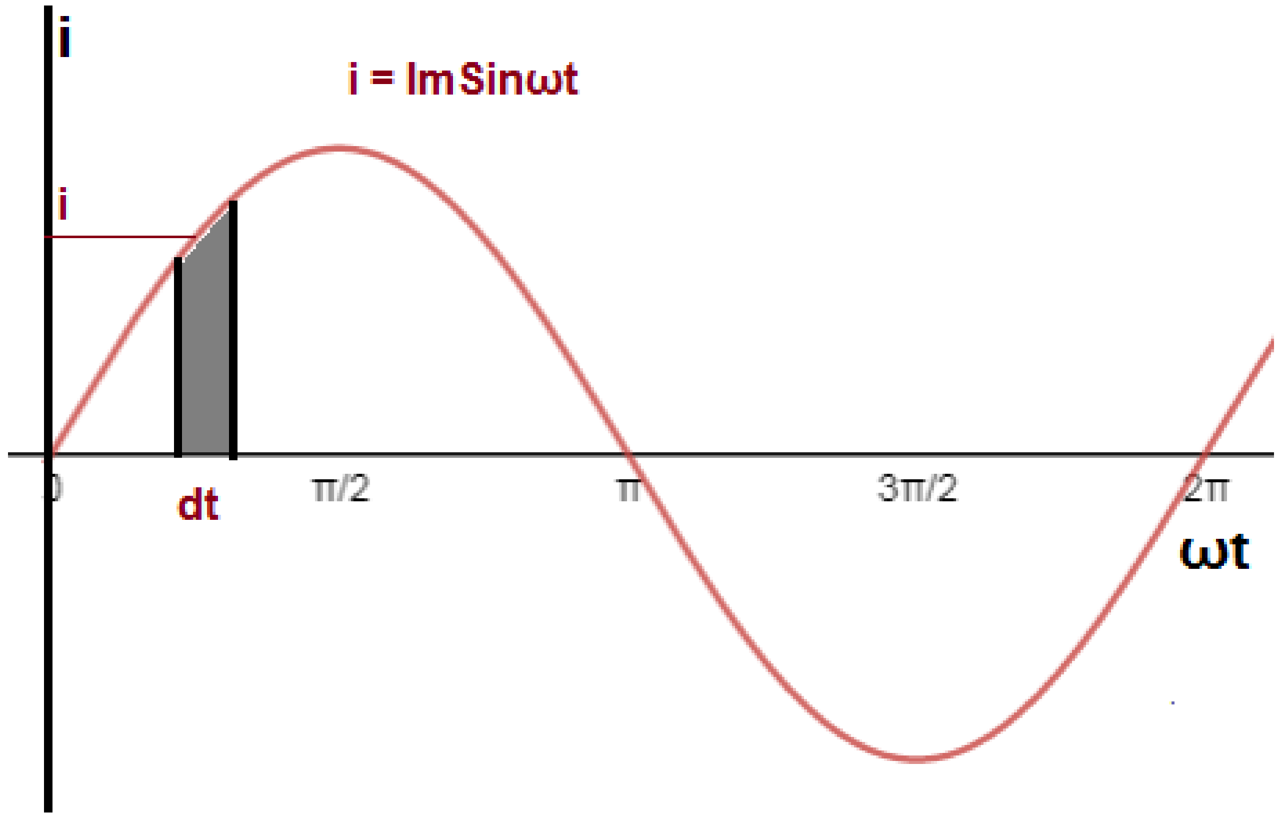

Fig1

$$\Rightarrow i_{av} = \frac{i_m}{\pi}[-cos\pi - (-cos0)]$$

$$\Rightarrow i_{av} = \frac{i_m}{\pi}[+1 - (-1)]$$

$$\Rightarrow i_{av} = \frac{2}{\pi} i_m = 0.637\ i_m \rightarrow (ii)$$

Similarly

$$v_{av} = \frac{2}{\pi} v_m = 0.637\ v_m \rightarrow (iii)$$

RMS value of alternating current:

RMS value of alternating current as that value of steady current which would generate the same amount of heat in a given resistance in given time as is done by AC current.

We know $i = i_m sin\omega t$

Again $dH = i^2 R dt \rightarrow (iv)$

$dH = {i_m}^2 sin^2\omega t R dt$

Now heat produced in time $\frac{T}{2}$ is given by

$$H = \int_0^{T/2} {i_m}^2 sin^2\omega t R dt$$

$$H = {i_m}^2 R \int_0^{T/2} sin^2\omega t dt$$

$$H = \frac{{i_m}^2 R}{2}\left[\frac{T}{2} - 0\right] \rightarrow (v)$$

The RMS represented by

$$H = {i_{rms}}^2 R \frac{T}{2} \rightarrow (vi)$$

Now from equation (v) and (vi), we get

$${i_{rms}}^2 R \frac{T}{2} = \frac{{i_m}^2 R}{2} \frac{T}{2}$$

$${i_{rms}}^2 = \frac{{i_m}^2}{2}$$

$$i_{rms} = \frac{i_m}{\sqrt{2}}$$

$$i_{rms} = .707\, i_m \rightarrow (vi)$$

Similarly

$$v_{rms} = .707\ v_m \rightarrow (vii)$$

Again the ratio of $RMS\ value$ and $Average\ value$ is known as $\boldsymbol{Form\ Factor}$

$$\frac{RMS\ value}{Avarage\ value} = \frac{\frac{v_m}{\sqrt{2}}}{\frac{2v_m}{\pi}} = \frac{\pi}{2\sqrt{2}} = 1.11 \rightarrow (viii)$$

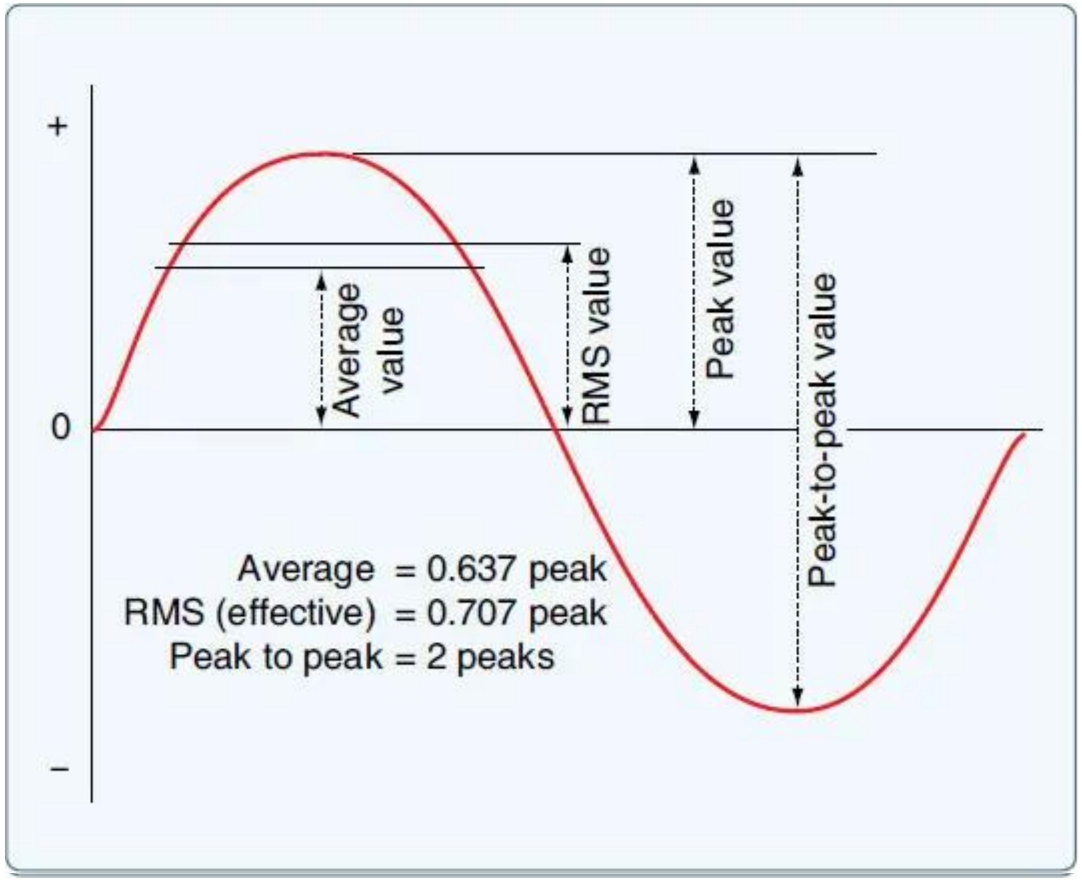
+
0
−
Average value
RMS value
Peak value
Peak-to-peak value
Average = 0.637 peak
RMS (effective) = 0.707 peak
Peak to peak = 2 peaks

Fig 2

Kirchhoff's Law:

Kirchhoff's law states that adding together voltage drops over different electric elements in any closed loop in a circuit should always add up to zero. The Kircchoff's rule states that as charge can not accumulated at any point in the circuit the balance of current requires that any incoming current at any point is exactly balance by outgoing current. AC circuit differ from DC circuit is that power supplied by the power source is not constant but change with time.

$$v = v_m sin(\omega t + \emptyset)$$

$$i = i_m sin(\omega t + \emptyset)$$

In DC circuit voltage is constant but in AC circuit voltage is sinusoidal function. In DC circuit voltages simply added, but in case AC circuit such voltage have to be added by sine function.

In case of AC circuit voltage is represented by vector phasor and not numbers, so voltage in AC circuit add by vector, not by numbers. Phasor are not real physical objects, these are just mathematical tools used to describe and add sinusoidal voltages in AC circuit in simpler way. In case of AC voltage it is a sine function one about care of phase Ø. In this case voltage represent as phasor having length ν and direction Ø. Addition of sinusoidal voltages in AC circuit then can be correctly performed by adding such vector phasor.

In AC circuit the same logic as in Kirchhoff's rule can be applied to state that the sum of voltages over any loop should be zero. However such sum is a vector sum of the voltage phasors and zero is a zero vector.

Such addition of voltages in AC circuit is usually represented by phasor diagram. In a phasor diagram the voltage phasors are drawn as vector relative to a phasor relative to a current in the circuit which is drawn at a fixed position. It is imagined that as the current phasor rotates with time the entire diagram rotate together with it.

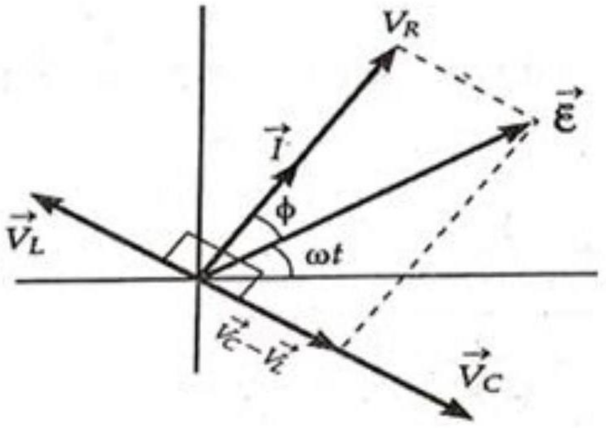

Fig 3

The KCL and KVL are

$$i_1 + i_2 + i_3 \ldots\ldots\ldots = 0 \rightarrow (ix)$$
$$v_1 + v_2 + v_3 \ldots\ldots\ldots = 0 \rightarrow (x)$$

This law are true for time domain and also frequency domain.

Series LCR Circuit:

LCR circuit is also known as a tuned or resonant circuit. It refers to an electrical circuit that comprises an $inductor(L)$, a $capacitor(C)$ and a $resistor(R)$, all are connected in series due to which same amount of current flow in the circuit.

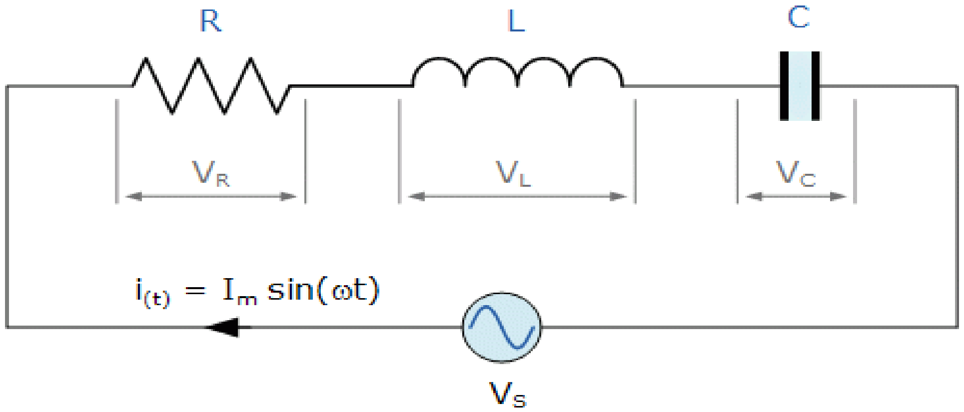

Fig 4

In this case the basic passive components resistance, capacitance and inductance have different phase relationship to each other when connected to a sinusoidal alternating voltage. In pure $R - circuit$ voltage wave form are in phase with the current. In pure $L - circuit$ voltage wave form leads the current by 90^0. In pure $C - circuit$ voltage wave form lags the current by 90^0. The phase difference depends upon the reactive value of the components being used and denoted by X, known as $\boldsymbol{Reactance}$ and it is zero, if the circuit element is resistive, positive if the circuit element is inductive and negative if it is capacitive leads to give another term known as $\boldsymbol{Impedance}$. Actually impedance is the addition of resistance and reactance for $LCR\ circuit$.

Circuit Element	Resistance(R)	Reactance(X)	Impedance(Z)
Resistor	R	$X_R = 0$	$Z_R = R$ $R < 0°$
Inductor	0	$X_L = \omega L$	$Z_L = j\omega L$ $Z_L = \omega L$ $< +90°$
Capacitor	0	$X_C = \frac{1}{\omega C}$	$Z_C = \frac{1}{j\omega C}$ $Z_C = \frac{1}{\omega C}$ $< -90°$

Table-1

In case of series $LCR\ circuit$ inductive reactance(X_L) and capacitive reactance(X_C) are the function of supply frequency the sinusoidal response of series $LCR\ circuit$ will therefore very with frequency. Therefore the individual voltage drops across each circuit element $R, C\ and\ R$, each will be out of phase with each other.

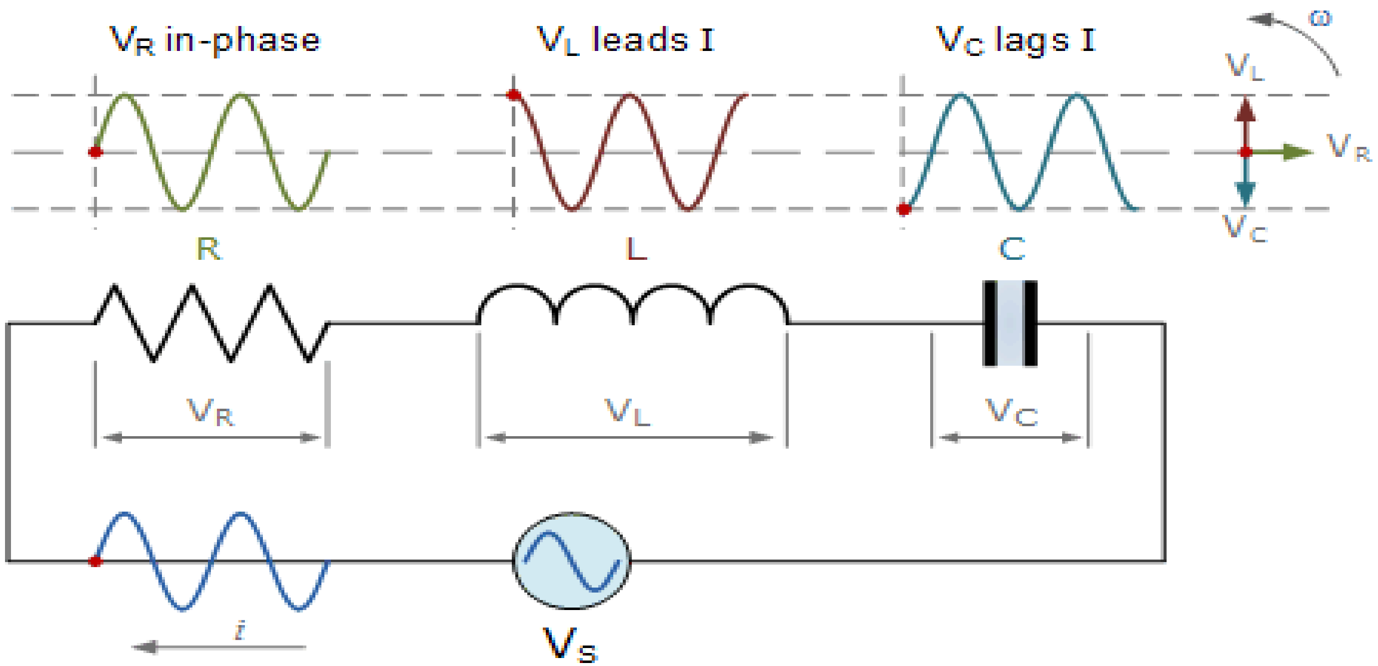

Fig 5

The amplitude of the voltage across the three component in series $LCR\ circuit$ is made up of three individual components voltages V_R, V_L, V_C with the current common to all three components. Because of phase difference of we can not add V_R, V_L, V_C directly instead these components must be added by phasor or vector method.

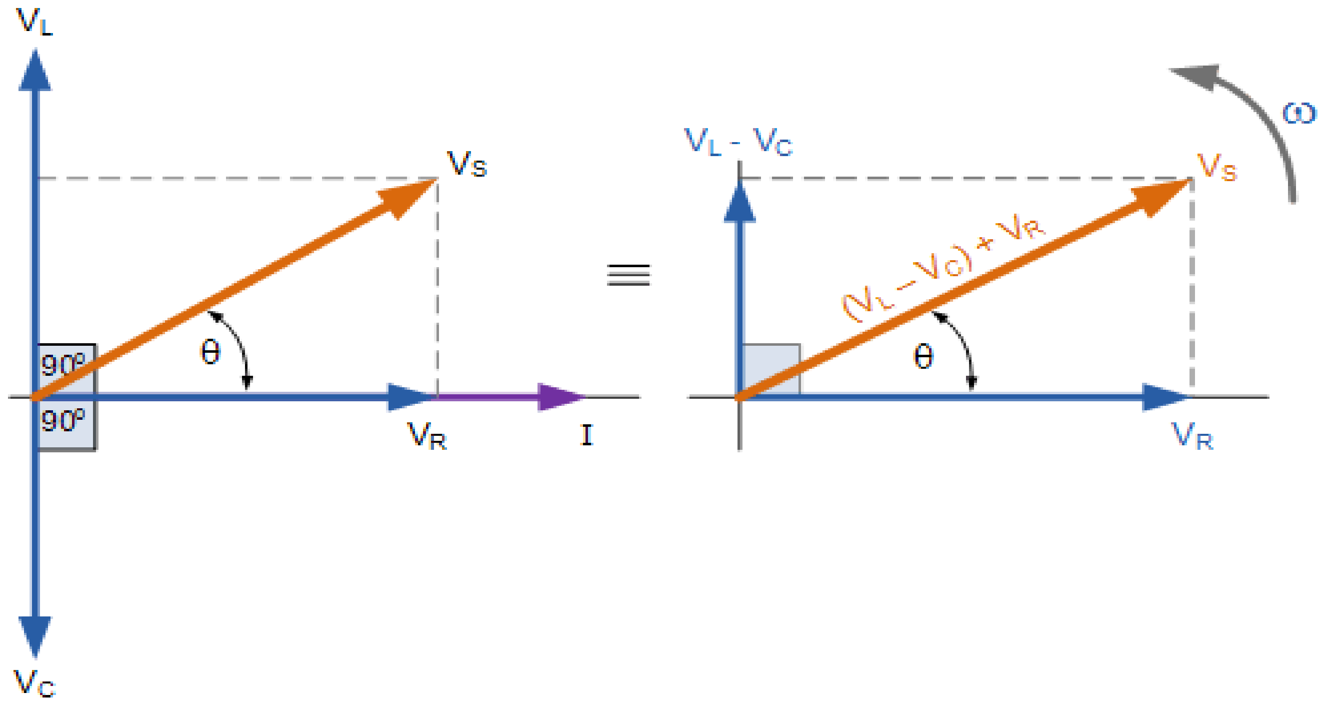

Fig 6

The total voltage drop is

$$V_s^2 = V_R^2 + (V_L - V_C)^2 \rightarrow (xii)$$

$$V_s = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Again we know that

$$V_R = iRsin(\omega t + 0°) = iR$$

$$V_L = iX_L sin(\omega t + 90°) = iX_L = iL\omega$$

$$V_C = iX_C sin(\omega t - 90°) = iX_C = i\frac{1}{C\omega}$$

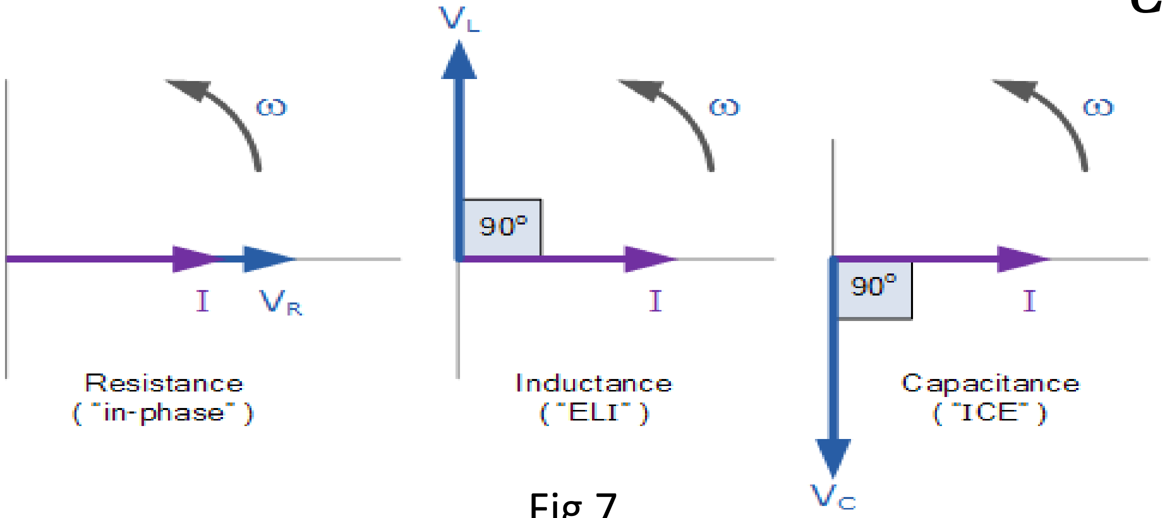

Fig 7

Therefore we can write

$$V_s = \sqrt{(iR)^2 + (iX_L - iX_C\,)^2} \rightarrow (xiii)$$

Again the total voltage $V_s = i.Z$, where Z is known as $\boldsymbol{impedance}$.

$$iZ = i\sqrt{R^2 + (X_L - X_C\,)^2}$$

$$Z = \sqrt{R^2 + (X_L - X_C\,)^2}$$

$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2} \rightarrow (xiv)$$

Here we take $L\omega > \frac{1}{C\omega}$

If $\frac{1}{C\omega} > L\omega$, then

$$Z = \sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2} \rightarrow (xv)$$

This impedance Z depends on angular frequency ω , X_L, X_C. If $X_L > X_C$, then overall circuit reactance is inductive giving the series circuit is lagging phase angle. If $X_C > X_L$, then the overall circuit reactance is capacitive giving the leading phase angle.

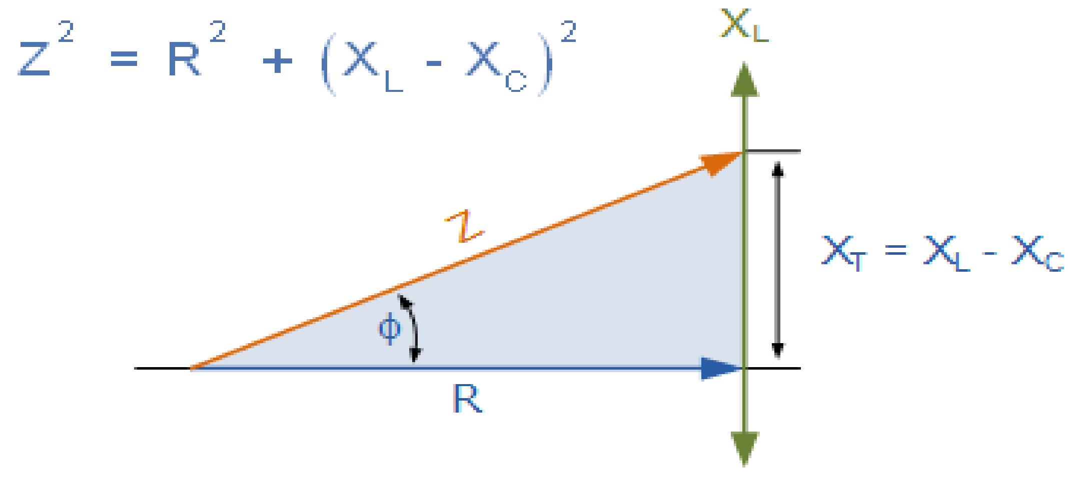

Fig 8