Another important point to be observed is that the set: for u & v as obtained above are valid for $\frac{U_\infty x}{\nu} >> 1$. In the neighbourhood of the leading edge at $x=0$, the B.L. approximation fails and consequently the set (soln.) are not valid therefore the improved approximations are necessary [L. Rocesslard].

<u>Skin Friction:</u>

The frictional force per unit area of the plate at distance x from the leading edge is called shearing stress at the wall.

In the case of flow past a flat plate

$$\sigma_W = (\sigma_{xy})_{y=0} = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu U_\infty f''(0)\sqrt{\frac{U_\infty}{\nu x}}$$

$$\sigma_W = \mu U_\infty \alpha \sqrt{\frac{U_\infty}{\nu x}}, \quad \text{where } \alpha = f''(0) = 0.332$$

$$= \rho \nu U_\infty \alpha \frac{1}{\sqrt{\frac{\nu x}{U_\infty} \times \frac{U_\infty^2}{\nu^2}}} \alpha \frac{U_\infty}{\nu}$$

$$= \rho U_\infty^2 \alpha \frac{1}{\sqrt{\frac{U_\infty x}{\nu}}} = \rho U_\infty^2 (0.332)\frac{1}{\sqrt{R_L}}$$, where R_L is the Reynolds no.

Hence the dimensionless shearing stress which is also known as local skin friction coefficient is given by

$$C_f = \frac{\sigma_W}{\rho U_\infty^2/2} = \frac{0.664}{\sqrt{R_L}}$$

The drag extend on the two sides of a plate of length l width b is given by

$$D = 2b\int_{x=0}^{l} (\sigma_{xy})_{y=0}\, dx$$

$$= 4\alpha b\rho \left(U_\infty^3 l \nu\right)^{1/2}$$

$$= 1.328\, b\rho \left(R_L \nu U_\infty^2 \nu\right)^{1/2}$$

$$= 1.328\, b\rho R_L^{\frac{1}{2}} \nu U_\infty$$

Hence a dimensionless drag coefficient

$$C_D = \frac{\text{Drag}}{\frac{1}{2}\rho A U_\infty^2},$$ where $A = 2bl$ denotes the total wetted surface area.

Hence,
$$C_D = \frac{1.328\, b\rho R_L^{\frac{1}{2}} \nu U_\infty}{\frac{1}{2}\rho \cdot 2bl U_\infty^2}$$

$$= \frac{1.328\, R_L^{\frac{1}{2}}}{R_L}$$

$$= 1.328\, R_L^{-\frac{1}{2}}.$$

Note: The similarity solution is also called the exact solution of B.L. equation.

§: Blasius Solution:

The problem of the B.L. flow along a thin flat plate at zero incidence in a uniform stream was first discovered by Blasius (1908).

Blasius solution of the B.L. equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty \frac{\partial U_\infty}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \longrightarrow (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \longrightarrow (2)$$

subject to the boundary condition

$$\left.\begin{array}{ll} u = v = 0 & \text{at } y = 0 \\ u = U_\infty & \text{at } y \to \infty \end{array}\right\} \longrightarrow (3)$$

donot differe in sense from the similarity solution. The equation (2) can be integrated by introducing a stream function $\psi(x, y)$ s.t.

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}.$$

Blasius solution: for the stream function is of the form

$$\psi = \sqrt{U_\infty \nu x}\, f(\eta)$$

where f is non-dimensional function. In the variable η is defined by $\eta = \sqrt{\frac{U_\infty}{\nu x}}\, y$

Blasius look the principle of similarity for velocity profile in the B.L. as

$$\frac{U}{U_\infty} = \phi\left(\frac{y}{\delta}\right), \quad \text{where } \delta(x) \sim \sqrt{\frac{\nu x}{U_\infty}}.$$

Now, substituting the values of $u, v, \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial^2 u}{\partial y^2}$ etc. in (1) we get the differential equation

$$ff'' + 2f''' = 0 \longrightarrow (4) \quad (\text{Usually known as Blasius equation})$$

subject to boundary condition

$$\left.\begin{array}{ll} f = f' = 0 & \text{at } \eta = 0 \\ f' = 1 & \text{at } \eta \to \infty \end{array}\right\} \longrightarrow (5)$$

Blasius obtained the set of (4) in the form of a series expansion for η small and an analytic expansion for η large, the two forms being matched at a suitable value of η.

Series solution for η small:

Since $f = f' = 0$ at $\eta = 0$, a series for $f(\eta)$ is

$$f(\eta) = c_2 \frac{\eta^2}{\lfloor 2} + c_3 \frac{\eta^3}{\lfloor 3} + c_4 \frac{\eta^4}{\lfloor 4} + c_5 \frac{\eta^5}{\lfloor 5} + \cdots + c_n \frac{\eta^n}{\lfloor n} + \cdots$$

$$f'(\eta) = c_2 \eta + c_3 \frac{\eta^2}{\lfloor 2} + c_4 \frac{\eta^3}{\lfloor 3} + c_5 \frac{\eta^4}{\lfloor 4} + \cdots$$

$$f''(\eta) = c_2 + c_3 \eta + c_4 \frac{\eta^2}{\lfloor 2} + c_5 \frac{\eta^3}{\lfloor 3} + \cdots$$

$$f'''(\eta) = c_3 + c_4 \eta + c_5 \frac{\eta^2}{\lfloor 2} + \cdots$$

Substituting these in the Blasius equation, we get,

$$0 = 2c_3 + 2c_4 \eta + (c_2^2 + 2c_5) \frac{\eta^2}{\lfloor 2} + (2c_6 + 4c_2 c_3) \frac{\eta^3}{\lfloor 3} + \cdots$$

Now, comparing the coefficients of different powers of η on both sides, we get,

$$c_3 = 0,\ c_4 = 0,\ c_5 = -\frac{1}{2} c_2^2,\ c_6 = 0,\ c_7 = 0,\ c_8 = -\frac{11}{21} c_2 c_5$$

All the non-zero coefficients ~~are~~ are found to depend only on c_2.

Now, obviousely, $c_2 = f''(0) = \alpha$ (say)

For small η,

$$f(\eta) = c_2 \frac{\eta^2}{\lfloor 2} + c_3 \frac{\eta^3}{\lfloor 3} + c_4 \frac{\eta^4}{\lfloor 4} + \cdots$$

$$= \frac{\eta^2}{1 \cdot \lfloor 2} \alpha - \frac{\eta^5}{2} \cdot \frac{\alpha^2}{\lfloor 5} + \frac{\eta^8}{2^2} \cdot \frac{11 \cdot \alpha^3}{\lfloor 8} - \cdots$$