ρ_{13} uniquely from the relation $\rho_{13} = \rho_{23} + \rho_{12}$ with a given value of ρ_{12}.

2017 (10) Hence the problem is solvable. This solution of the three body problem is known as the straight line soln.

case II :- when the mutual distances between the masses expand or contract in the same ratio so that the geometrical configuration remains invariant.

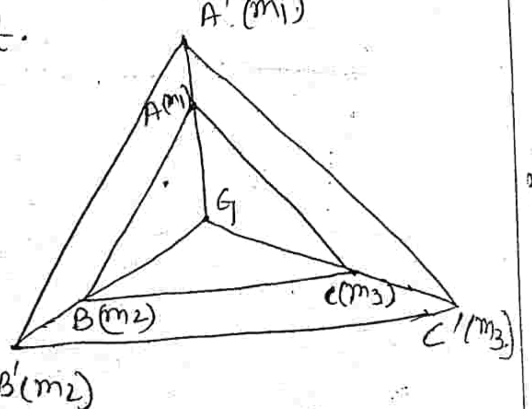
In this case –

$$\frac{\rho_{12}}{(\rho_{12})_0} = \frac{\rho_{23}}{(\rho_{23})_0} = \frac{\rho_{31}}{(\rho_{31})_0} = \phi(t); \text{ say} \to (1)$$

where $\phi(t)$ is a function of t only and $(\rho_{ij})_0$ is the distance between the ith and jth mass at time $t=0$. Under the same consideration that $\vec{r}_i$ $(i=1,2,3)$ is the position vector of m_i with respect to the centre of mass. Then –

$$\frac{r_1}{(r_1)_0} = \frac{r_2}{(r_2)_0} = \frac{r_3}{(r_3)_0} = \phi(t) \to (2)$$

where each ratio is equal to the same proportional constant (but function of time) $\phi(t)$ due to the similarity of the triangle formed by the masses at time $t=0$ & at time t.



Again, since the angle between r_i and r_j is the same as $(r_i)_0$ and $(r_j)_0$, therefore the masses rotate with constant angular velocity about the centre of mass in their plane of projection.

$$\frac{1}{r_i}\frac{d}{dt}(r_i^2\dot{\theta}) = \frac{1}{r_i}\frac{d}{dt}\left[(r_i)_0^2\,\phi^2(t)\dot{\theta}\right], \text{ using } ②.$$

$$= \frac{1}{r_i}(r_i)_0^2\frac{d}{dt}\left[\phi^2(t)\,\dot{\theta}\right] \rightarrow ③$$

Now, by the principle of conservation of angular momentum of the three body, we have

$$\sum_{i=1}^{3}\vec{r_i}\times m_i\vec{v_i} = \text{a constant vector} = \vec{L}\ (\text{say})$$

$$\Rightarrow \sum_{i=1}^{3} r_i\,\hat{r_i}\times m_i r_i\,\dot{\theta_i}\,\hat{\theta_i} = \vec{L}$$

$$\Rightarrow \sum_{i=1}^{3} m_i r_i^2 \dot{\theta}_i \, (\hat{r}_i \times \hat{\theta}_i) = \vec{L}$$

$$\Rightarrow \sum_{i=1}^{3} m_i r_i^2 \dot{\theta}_i \, \hat{n} = \vec{L} \; ; \text{ where } \hat{n} = \hat{r}_i \times \hat{\theta}_i$$

$$\Rightarrow \sum_{i=1}^{3} m_i r_i^2 \dot{\theta} \, \hat{n} = \vec{L} \quad [\because \dot{\theta}_i = \dot{\theta} = \text{constant}] -$$

$$\Rightarrow \left(\sum_{i=1}^{3} m_i r_i^2\right) \dot{\theta} \, \hat{n} = L\hat{n}.$$

$$\Rightarrow \dot{\theta} \left\{\sum_{i=1}^{3} m_i (r_i)_0^2 \right\} \phi^2(t) = L.$$

$$\Rightarrow \dot{\theta} \, I_0 \, \phi^2(t) = L \rightarrow (4)$$

where $I_0 = \sum_{i=1}^{3} m_i (r_i)_0^2$, be the moment of inertia of the system about the axis of rotation at time $t=0$.

$$(4) \Rightarrow \dot{\theta} \, \phi^2(t) = \frac{L}{I_0} = \text{a constant} = L_0 \text{ (say)} \rightarrow (5)$$

Hence $(3) \Rightarrow \frac{1}{r_i} \frac{d}{dt} (r_i^2 \dot{\theta}) = 0$

This shows that the cross radial component of acceleration of the three masses are zero. i.e the acceleration of the masses are purely radial. Hence the force f_i acting on the masses m_i pass through the centre of mass.

Now $m_i \ddot{\vec{r}}_i = \vec{F}_i = m_i (f_i)_r \hat{r}_i$

$$\Rightarrow \ddot{\vec{r}}_i = (f_i)_r \hat{r}_i = (f_i)_r \frac{\vec{r}_i}{r} \quad [\text{writing } \vec{f}_i = (f_i)_r \hat{r}_i]$$

$$= -n_i^2 \vec{r}_i, \rightarrow (6) \text{ where } \frac{(f_i)_r}{r} = -n_i^2 \; (i=1,2,3)$$

The eqn (6) is smillilar to the case I. Hence the problem. is solvable.

Both ... to the equi lateral