Application of Demoivre's Theorem:

How De-moivre's Theorem works

$$(\cos\theta + i\sin\theta)^n = \underline{\cos n\theta + i\sin n\theta}$$

$(\cos\theta + i\sin\theta)^5 (\cos\theta - i\sin\theta)^3$

$= (\cos 5\theta + i\sin 5\theta)(\cos 3\theta - i\sin 3\theta)$

$= \underline{\cos 5\theta \cos 3\theta} - i\cos 5\theta . \sin 3\theta + i\sin 5\theta \cos 3\theta + \underline{\sin 3\theta . \sin 5\theta}$

$= \cos(5\theta - 3\theta) + i\sin(5\theta - 3\theta)$

$= \underline{\cos 2\theta + i\sin 2\theta}$

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How De-moivre's Theorem works

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

$$(1+i)^{100}, \quad x+iy = r\cos\theta + ir\sin\theta$$

$$1+i = r\cos\theta + ir\sin\theta \quad \text{——— (1)}$$

Equating Real and Imaginary part

$$\left.\begin{array}{l} r\cos\theta = 1 \\ r\sin\theta = 1 \end{array}\right\} \quad r^2\cos^2\theta + r^2\sin^2\theta = 1^2 + 1^2$$

$$r^2(\cos^2\theta + \sin^2\theta) = 2$$

$$r^2 = 2$$

$$\boxed{r = \sqrt{2}}$$

$$\left.\begin{array}{l} \sqrt{2}\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \\ \sqrt{2}\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{\sqrt{2}} \end{array}\right\}$$

$$\Rightarrow \boxed{\theta = \frac{\pi}{4} \text{ or } 45^\circ}$$

Now $1+i = r(\cos\theta + i\sin\theta)$

$$1+i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$(1+i)^{100} = \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^{100}$$

$$= 2^{\frac{1}{2}\times 100}\left[\cos\frac{100\pi}{4} + i\sin\frac{100\pi}{4}\right]$$

$$= 2^{50}\left[\cos 25\pi + i\sin 25\pi\right]$$

$$= 2^{50}\left[-1 + i0\right]$$

$$= -2^{50} \text{ Ans}$$

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