

Series Solution

Power Series

An infinite series of the form $\sum_{n=0}^{\infty} c_n (x-x_0)^n$

$$= c_0 + c_1(x-x_0) + c_2(x-x_0)^2 + \dots \rightarrow \textcircled{1}$$

is called a power series in $(x-x_0)$

The series converges absolutely for $|x| < R$

where $R = \lim_{n \rightarrow \infty} \left| \frac{c_n}{c_{n+1}} \right|$ provided the limit exists, R is said to be the radius of convergence of $\textcircled{1}$

Analytic function:

A function $f(x)$ defined on an interval containing the point $x=x_0$ is called analytic at x_0 if

its Taylor series $\sum_{n=1}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x-x_0)^n$ exists and

converges to $f(x)$ for all x in the interval of convergence $(-R, R)$