$$\frac{\partial}{\partial x}\int_0^\infty u(U-u)\,dy + \frac{dU}{dx}\int_0^\infty (U-u)\,dy = \frac{\sigma_0}{\rho} \longrightarrow (x)$$

for steady flow, under no pressure gradient, the equation (x) becomes,

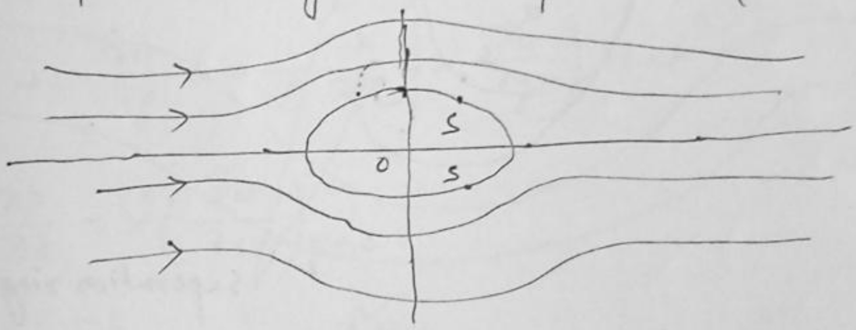
$$\frac{\partial}{\partial x}\int_0^\infty u(U-u)\,dy = \frac{\sigma_0}{\rho}$$

or, $$\sigma_0 = \rho\frac{\partial}{\partial x}\int_0^\infty u(U-u)\,dy.$$

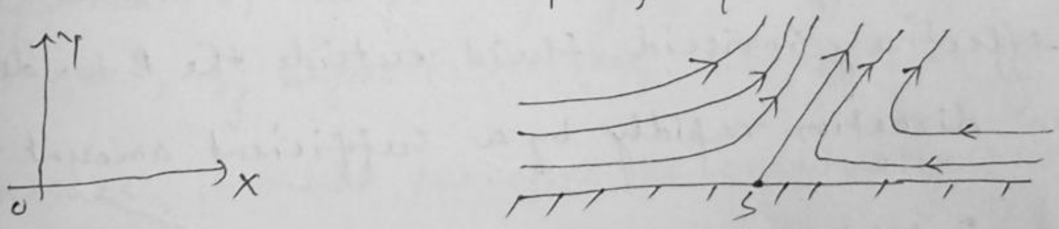
x

Figures Separation of Boundary layer:

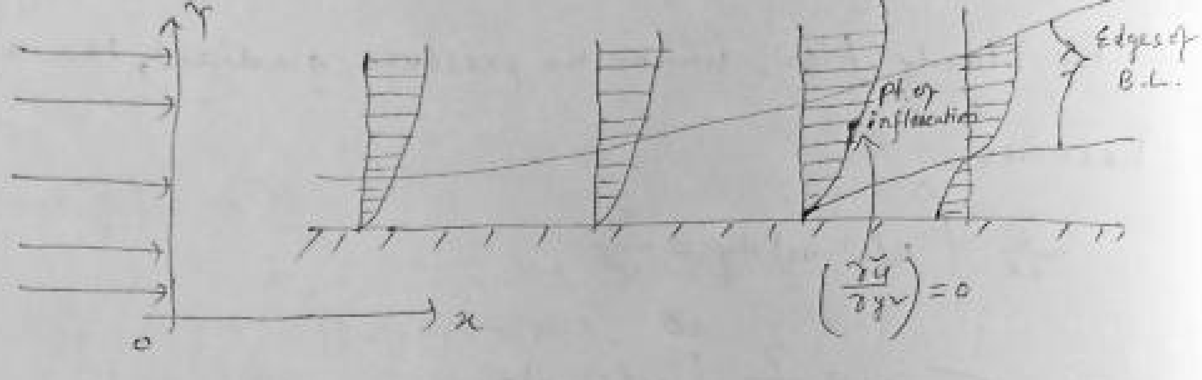
Fig(i): Flow past a body with separation ($S \to$ pt. of separation)



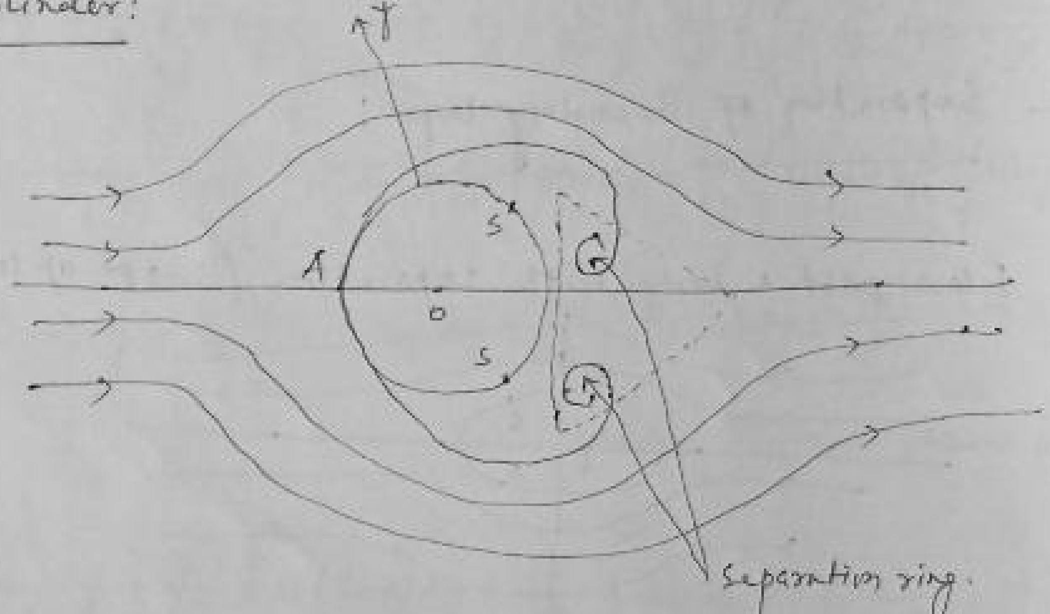
Fig(ii) Stream lines near the pt. of separation



(iii) The velocity distribution near the point of separation.



(iv) Boundary layer separation and vortex formation on a circular cylinder:



The separation of steady B.L. at a point plane or rounded ring wall without edges is observed to occure whenever the velocity of the effectively inviscid fluid outside the B.L. decreases in the flow direction rapidly by a sufficient amount.

Outside the B.L., decreases of velocity is associated with the increase of pressure. Since the pressure is approximately uniform across the B.L, so that the pressure gradient which produces the acceleration of the external stream acts equally in the fluid within the B.L.

In general, the fluid particles behind the point of separation follow the pressure gradient and move in a direction opposit to the external stream.

Analytical approach:

The point of separation is defined as a limit between forward and reverse flow in the layer in immediate neighbourhood of the wall. Thus if u be the velocity component in the direction tangential to the wall and y be the distance measured perpendicular to the wall, then the point of separation is given by $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$.

Thus, infact gives, the separation when the wall friction vanishes. Again, since at the wall u and v vanish, it follow from the B.L equation

$\left(i.e.\ u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}\right)$ that

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \nu\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0}$$

hence if $\frac{\partial p}{\partial x} > 0$, $\frac{\partial^2 u}{\partial y^2} > 0$ at $y=0$.

Now, upstream of the point of separation $\frac{\partial u}{\partial y} > 0$ at the wall and if $\frac{\partial p}{\partial x}$ is +ve, $\frac{\partial u}{\partial y}$ begins to increase. But outside the B.L. $\frac{\partial u}{\partial y}$ vanishes, it must therefore be eventually and hence $\frac{\partial^2 u}{\partial y^2}$ must vanish at some point. Therefore, in the B.L., the velocity profile has a point of inflexion, where $\frac{\partial^2 u}{\partial x^2} = 0$.

Generally, speaking, the B.L. equations are only valid upto the point of separation because downstream from the point of separation the B.L. becomes so thick that the assumption which were made in the derivation of the B.L. equations no longer apply.

Note: The point of separation is determined from the condition

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$$

1. If $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ then the flow is on the verge of separation.
2. If $\left(\frac{\partial u}{\partial y}\right)_{y=0} > 0$, then the flow will not separate or flow will remain attached within the surface.
3. If $\left(\frac{\partial u}{\partial y}\right)_{y=0} < 0$ then the flow has separated or this is the condition for detached flow.

Ex. For the following velocity profiles determine whether the flow has separated or on the verge of separation.

(i) $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

(ii) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right)^2 - \left(\frac{y}{\delta}\right)^3$

(iii) $\frac{u}{U} = -2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^3$

Solⁿ. (1) Given profile is

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

or $$u = \frac{3U}{2}\left(\frac{y}{\delta}\right) - \frac{U}{2}\left(\frac{y}{\delta}\right)^3$$

$$\therefore \frac{\partial u}{\partial y} = \frac{3U}{2}\cdot\frac{1}{\delta} - \frac{U}{2\delta^3}\cdot 3y^2$$

At $y=0$, $\frac{\partial u}{\partial y} = \frac{3U}{2}\cdot\frac{1}{\delta}$