The eq^n (6) is smililar to the case I. Hence the problem is solvable.

<u>Path of the masses corresponding to the equi lateral triangle solution:</u>

2018 (10) Let $\vec{r_1}$ be the possition vector of mass m_1 from centre of mass c of the mass m_i $(i=1,2,3)$ and $\vec{\rho_{ij}}$ be the position vector from ith mass to the jth mass: Then the eq^n of motion of mass m_1 is

$$\ddot{\vec{r_1}} = k^2 \left[\frac{m_2}{\rho_{12}^2} \frac{\vec{\rho_{12}}}{\rho_{12}} + \frac{m_3}{\rho_{13}^2} \frac{\vec{\rho_{13}}}{\rho_{13}} \right]$$

$$= k^2 \left[\frac{m_2}{\rho_{12}^3} \vec{\rho_{12}} + \frac{m_3}{\rho_{13}^3} \vec{\rho_{13}} \right] \rightarrow (1)$$

But for equi lateral triangle solutions

$$\rho_{12} = \rho_{23} = \rho_{31} = \rho \text{(say)}$$

$$\therefore (1) \Rightarrow \ddot{\vec{r_1}} = \frac{k^2}{\rho^3} \left[m_2 \vec{\rho_{12}} + m_3 \vec{\rho_{13}} \right] \rightarrow (2)$$

we have, $m_1 \vec{r_1} + m_2 \vec{r_2} + m_3 \vec{r_3} = 0$

$$\Rightarrow (m_1 + m_2 + m_3) \vec{r_1} - m_2 \vec{r_1} - m_3 \vec{r_1} + m_2 \vec{r_2} + m_3 \vec{r_3} = 0$$

$$\Rightarrow M \vec{r_1} + m_2 (\vec{r_2} - \vec{r_1}) + m_3 (\vec{r_3} - \vec{r_1}) = 0$$

$$\Rightarrow M \vec{r_1} + m_2 \vec{\rho_{12}} + m_3 \vec{\rho_{13}} = 0 \rightarrow (3)$$

where $M = m_1 + m_2 + m_3$ = total mass of the system.

and $\rho_{ij} = \vec{r_j} - \vec{r_i}$

② $\Rightarrow$ $\ddot{\vec{r_1}} = \frac{K^2}{\rho^3}(-M\vec{r_1})$, using ③ $\rightarrow$ ④

③ $\Rightarrow$ $m_2 \vec{\rho_{12}} + m_3 \vec{\rho_{13}} = -M\vec{r_1}$

$\Rightarrow (m_2 \vec{\rho_{12}} + m_3 \vec{\rho_{13}})^2 = M^2 r_1^2$

$\Rightarrow m_2^2 \rho_{12}^2 + m_3^2 \rho_{13}^2 + 2m_1 m_2 (\vec{\rho_{12}} \cdot \vec{\rho_{13}}) = M^2 r_1^2$

$\Rightarrow m_2^2 \rho^2 + m_3^2 \rho^2 + 2m_2 m_3 \rho\rho \cos 60^\circ = M^2 r_1^2$

$\Rightarrow \rho^2 (m_2^2 + m_3^2 + 2m_2 m_3 \cdot \frac{1}{2}) = M^2 r_1^2$

$\Rightarrow \rho^2 = \frac{M^2 r_1^2}{(m_2^2 + m_3^2 + m_2 m_3)}$

$\Rightarrow \rho = \frac{M r_1}{(m_2^2 + m_3^2 + m_2 m_3)^{1/2}}$

The eq^n ④ reduces to.

$$\ddot{\vec{r_1}} = -\frac{MK^2}{\left\{\frac{M r_1}{(m_2^2 + m_3^2 + m_2 m_3)^{1/2}}\right\}^3}\vec{r_1}$$

$$= -\frac{K^2 (m_2^2 + m_3^2 + m_2 m_3)^{3/2}}{M^2 r_1^3}\vec{r_1}$$

$$= -\frac{K^2 M_1}{r_1^2} \cdot \frac{\vec{r_1}}{r_1} \quad ; \quad M = \frac{(m_2^2 + m_3^2 + m_2 m_3)^{3/2}}{M^2}$$

$$= -\frac{K^2 M_1}{r_1^2} \hat{r_1} \quad \rightarrow ⑤$$

The eq^n (5) is exactly similar to the eq^n of motion for a two body problem under inverse square law.

Hence the motion of mass m_1 reduces to a motion similar to the motion of two body problem as if it was unit mass under the attraction of M_1 and were placed at the centre of mass.

Similar results can be found for the other two masses ~~Hence~~ Hence the orbit of the masses is a conic section i.e an ellipse, parabola, hyperbola depending upon the initial velocity.