As $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is +ve hence the flow will not separated or flow will remain attached with surface.

(ii) Given, $u = 2U\left(\frac{y}{\delta}\right)^2 - U\left(\frac{y}{\delta}\right)^3$

$$\therefore \frac{\partial u}{\partial y} = \frac{4U}{\delta^2}y - \frac{3U}{\delta^3}y^2$$

$\therefore \left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$ i.e. the flow is on the verge of separation.

(iii) Given $u = -2U\left(\frac{y}{\delta}\right) + U\left(\frac{y}{\delta}\right)^2$

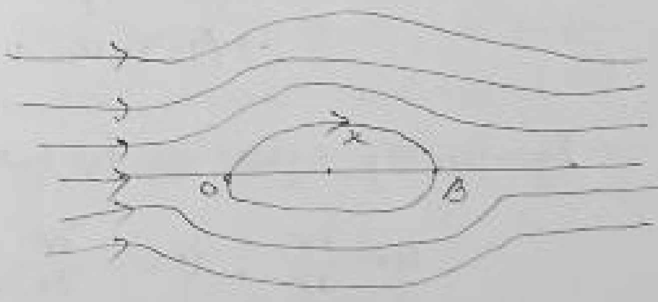
$$\therefore \left(\frac{\partial u}{\partial y}\right)_{y=0} = -\frac{2U}{\delta},$$

Since $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is −ve, therefore the flow has separated.

—x—

§: Blasius solution for flow past a cylinder: symmetrical case:

[By the symmetrical case, we mean that the cylinder is symmetrical about an axis which is parallel to the stream at a large distance from it.]



Let us consider the steady B.L. equations past a cylinder. The B.L. equations are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \longrightarrow (i)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \longrightarrow (ii)$$

Subject to the boundary condition

$$\left.\begin{array}{ll} u = v = 0 & \text{at } y = 0 \\ u = U & \text{at } y \to \infty \end{array}\right\} \longrightarrow (iii)$$

where x is measured along the contour from the forward stagnation point and y is measured perpendicular to the contour as shown in figure.

In Blasius method, the velocity of the potential flow is assumed to have the form of a power series in x. Let the potential flow is given by the series

$$U(x) = u_1 x + u_3 x^3 + u_5 x^5 + u_7 x^7 + \cdots \quad (\text{iv})$$

Here the coefficients $u_1, u_3, u_5, \cdots$ etc depend only on the shape of the body and are compared to be known. Hence the pressure term in equation (i) becomes,

$$U\frac{dU}{dx} = (u_1 x + u_3 x^3 + u_5 x^5 + \cdots)\frac{d}{dx}(u_1 x + u_3 x^3 + u_5 x^5 + \cdots)$$

$$= (u_1 x + u_3 x^3 + u_5 x^5 + \cdots)(u_1 + 3u_3 x^2 + 5u_5 x^4 + \cdots)$$

$$= u_1^2 x + 3u_1 u_3 x^3 + (6u_1 u_5 + 3u_3^2)x^5 + (8u_1 u_7 + 8u_3 u_5)x^7$$
$$+ (10u_1 u_9 + 10u_3 u_7 + 5u_5^2)x^9 + \cdots$$
$$+ (12u_1 u_{11} + 12u_3 u_9 + 12u_5 u_7)x^{11} + \cdots$$

In view of equation (ii), we take,

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

where ψ is the stream function.

We define $\eta = y\left(\frac{u_1}{\nu}\right)^{\frac{1}{2}}$

$$\eta = \frac{y}{\delta}, \quad \delta = \sqrt{\frac{\nu x}{U}}$$
$$= y\sqrt{\frac{U}{\nu x}}$$
$$= y\sqrt{\frac{u_1 x}{\nu x}} = y\sqrt{\frac{u_1}{\nu}}$$

[Fluid dynamics and stream function,

$$\psi = \sqrt{\frac{\nu}{u_1}}\left[u_1 x f_1(\eta) + u_1 u_3 x^3 f_3(\eta) + 6u_5 x^5 f_5(\eta) + \nu u_7 x^7 f_7(\eta) + 10u_9 x^9 f_9(\eta) + 12u_{11} x^{11} f_{11}(\eta) + \cdots\right]$$

In order to present the functional coefficients f_1, f_3, f_5, independent of the particular properties of the profiles $u_1, u_3, \cdots$ it is necessary to split them up as follows:

$$f_5 = g_5 + \frac{u_3^2}{u_1 u_5} h_5$$

$$f_7 = g_7 + \frac{u_3 u_5}{u_1 u_7} h_7 + \frac{u_3^3}{u_1^2 u_7} k_7$$

$$f_9 = g_9 + \frac{u_3 u_7}{u_1 u_9} h_9 + \frac{u_5^2}{u_1 u_9} k_9 + \frac{u_3^2 u_5}{u_1^2 u_9} j_{11} + \frac{u_3^4}{u_1^3 u_9} g_9 \quad \text{and so on.}$$

Now, we obtain,

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}$$

$$= \left[u_1 x f_1' + 4 u_3 x^3 f_3' + 6 u_5 x^5 f_5' + 8 u_7 x^7 f_7(\eta) + \cdots \right]$$

and $$v = -\frac{\partial \psi}{\partial x} = -\sqrt{\frac{\nu}{u_1}} \left[u_1 f_1 + 12 u_3 x^2 f_3 + 30 u_5 x^4 f_4 + \cdots \right]$$

where the primes denote differentiation w.r. to η.

Now, calculating $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial^2 v}{\partial y^2}$ and substituting them in (IV) where $U \frac{dU}{dx}$ is replaced by its equivalent expansion and equating the coefficients of like powers of x, we obtain a system of ordinary differential equation for the functionals $f_1, f_3, \cdots$ etc. The first four of these eq^{ns} are

$$\left.\begin{aligned}
&f_1'^2 - f_1 f_1'' = 1 + f_1''' \\
&4 f_1' f_3' - 3 f_1'' f_3 - f_1 f_3'' = 1 + f_3''' \\
&6 f_1' g_5' - 5 f_1'' g_5 - f_1 g_5'' = 1 + g_5''' \\
&6 f_1' h_5' - 5 f_1'' h_5 - f_1 h_5'' = \tfrac{1}{2} h_5''' - 8\left(f_3'^2 - f_3 f_3''\right)
\end{aligned}\right\} \quad \text{(V)}$$

In the above set of equations, only the first equation is non-linear and others are all linear. The first equation is the same as that Hiemenz equation.

The boundary conditions for $f_1, f_2, \ldots$ are

$$\left.\begin{aligned} &f_1 = f_1' = 0\ ;\ f_3 = f_3' = 0,\ g_5 = g_5' = 0,\ h_5 = h_5' = 0 \text{ at } \eta = 0 \\ &f_1' = 1,\ f_3' = \tfrac{1}{4},\ g_5' = \tfrac{1}{6},\ h_5' = 6 \quad \text{at } \eta \to \infty \end{aligned}\right\} \to (\text{VI})$$

The functions $f_1\, f_3$ have been evaluated by Hiemenz (1911). The graphs for f_1' and f_3' are shown below.

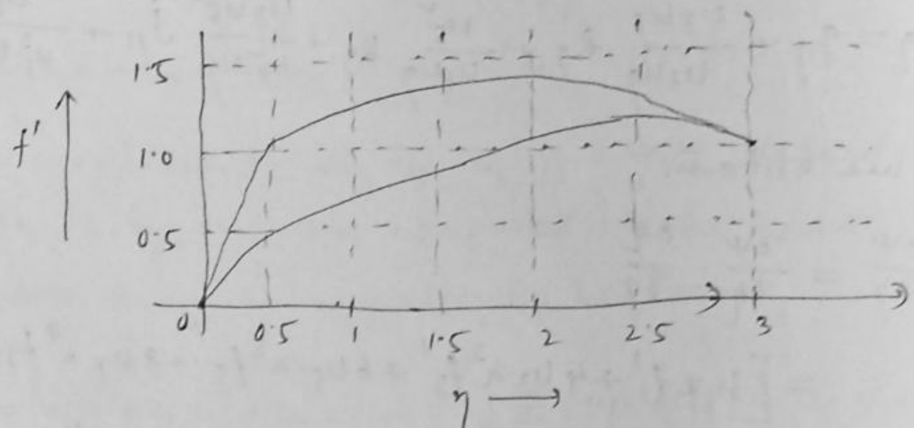
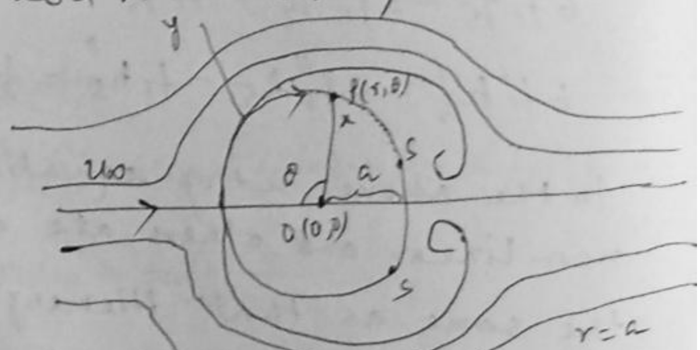


Fig: The functions f_1' and f_3' which appear in the Blasius power series.

L. Howarth improved the tables for f_3 and the functions g_5 and h_5 have later been evaluated by N. Froessling. Later A. Alrich considerably extended the scope of the calculations having evaluated them upto and including the 9^{th} power in x. A.N. Tifford calculated the functions associated with term x'' and improved the occuring of the existing tables.

§: Application of Blasius method in the case of a circular cylinder:



Let us consider the steady 2-D flow past a circular cylinder.

The Boundary layer eq^n are.