$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \longrightarrow (i)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \longrightarrow (ii)$$

subject to the boundary conditions

$$\left.\begin{array}{ll} u = v = 0 & \text{at } y = 0 \\ u = U(x) & \text{at } y \to \infty \end{array}\right\} \longrightarrow (iii)$$

Let 'a' be the radius of the circular cylinder. We use polar coordinates (r, θ) with the pole at the centre of the cylinder and the initial line against the direction of the free stream velocity U_∞. We know by ideal fluid theory, the radial and cross-radial components of velocity in irrotational flow past a cylinder are given by

$$\left.\begin{array}{l} q_r = -U_\infty\left(1 - \frac{a^2}{r^2}\right)\cos\theta \\ q_\theta = U_\infty\left(1 + \frac{a^2}{r^2}\right)\sin\theta \end{array}\right\} \longrightarrow (iv)$$

Since the B.L. developed on the cylinder is thin, therefore inside the B.L. $r \simeq a$.

Then a writting $x = a\theta$ so that x measures the distance on the circumference from the forward stagnation point A. Following Blasius, the ideal fluid velocity distribution just outside the B.L. can be take and simply given by

$$q_\theta = U(x) = 2U_\infty \sin\theta$$

$$= 2U_\infty \sin\left(\frac{x}{a}\right)$$

Expanding the sine function, we have,

$$U(x) = 2U_\infty\left[\frac{x}{a} - \frac{1}{\underline{|3}}\left(\frac{x}{a}\right)^3 + \frac{1}{\underline{|5}}\left(\frac{x}{a}\right)^5 - \cdots\right]$$

$$= u_1 x + u_3 x^3 + u_5 x^5 + u_7 x^7 + \cdots$$

where $u_1 = \frac{2U_\infty}{a}$, $u_3 = -\frac{2U_\infty}{\lfloor 3\, a^3}$, $u_5 = \frac{2U_\infty}{\lfloor 5\, a^5}$, ...

Since U_∞ and a are known therefore these u's are known.

We define, $\eta = \left(\frac{u_1}{\nu}\right)^{1/2} = \frac{y}{a}\left(\frac{2U_\infty a}{\nu}\right)^{\frac{1}{2}}$

$$= \frac{y}{a}\sqrt{2R}$$

where $R = \frac{U_\infty a}{\nu}$ is the Reynolds number.

We also define

$$u = \frac{\partial \psi}{\partial y}, \text{ and } v = -\frac{\partial \psi}{\partial \eta}$$

where the stream function ψ is given by

$$\psi = \nu\sqrt{2R}\left[\left(\frac{x}{a}\right)f_1 - \frac{4}{\lfloor 3}\left(\frac{x}{a}\right)^3 f_3 + \frac{6}{\lfloor 5}\left(\frac{x}{a}\right)^5 f_4 - \cdots\right]$$

$$\therefore u = \frac{\partial \psi}{\partial y} = 2U_\infty\left[\left(\frac{x}{a}\right)f_1' - \frac{4}{\lfloor 3}\left(\frac{x}{a}\right)^3 f_3' + \frac{6}{\lfloor 5}\left(\frac{x}{a}\right)^5 f_5' - \frac{8}{\lfloor 7}\left(\frac{x}{a}\right)^7 f_7 + \cdots\right]$$

with $f_5' = g_5' + \frac{u_3^2}{u_1 u_5} h_5' = g_5' + \frac{10}{3} h_5'$

$$f_7' = g_7' + \frac{u_3 u_5}{u_1 u_7} h_7' + \frac{u_3^3}{u_1^2 u_7} k_7' = g_7 + 7h_7' + \frac{70}{3} k_7' \text{ and so on.}$$

Here f's are functions of η only.

The shearing stress at the cylinder at a distance x from the forward stagnation point is given by

$$\sigma_0 = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \rho\nu\frac{\sqrt{2R}}{a}\left(\frac{\partial u}{\partial \eta}\right)_{\eta=0}$$

$$= \rho\nu\frac{\sqrt{2R}}{a}\cdot 2U_\infty\left[\left(\frac{x}{a}\right)f_1''(0) - \frac{4}{\lfloor 3}\left(\frac{x}{a}\right)^3 f_3''(0) + \frac{6}{\lfloor 5}\left(\frac{x}{a}\right)^5 f'''(0) - \cdots\right]$$

$$\sigma_0 = \frac{1}{2}\rho\nu\frac{\sqrt{R}}{a}U_\infty\left[4\sqrt{2}\left(\frac{x}{a}\right)f_1''(0) - \frac{16\sqrt{2}}{\lfloor 3}\left(\frac{x}{a}\right)^3 f_3''(0) + \frac{24\sqrt{2}}{\lfloor 5}\left(\frac{x}{a}\right)^5 f'''(0) - \cdots\right]$$

$$= \frac{1}{2}\rho U_\infty^2 \times \frac{\nu}{U_\infty a}\sqrt{R}\left[4\sqrt{2}\left(\frac{x}{a}\right)f_1''(0) - \frac{16\sqrt{2}}{\lfloor 3}\left(\frac{x}{a}\right)^3 + \frac{24\sqrt{2}}{\lfloor 5}\left(\frac{x}{a}\right)^5 f_5'''(0) - \cdots\right]$$

$$= \frac{1}{2}\rho U_\infty^2 R^{-\frac{1}{2}}\left[4\sqrt{2}\left(\frac{x}{a}\right)f_1''(0) - \frac{16\sqrt{2}}{\lfloor 3}\left(\frac{x}{a}\right)^3 f_3''(0) + \frac{24\sqrt{2}}{\lfloor 5}\left(\frac{x}{a}\right)^5 f_5'''(0) - \cdots\right]$$

But it is found that

$$f_1'''(0) = 1.2326, \quad f_3''(0) = 0.7244$$

$$\therefore \sigma_0 = \frac{1}{2}\rho U_\infty^2 R^{-\frac{1}{2}}\left[6.973\left(\frac{x}{a}\right) - 2.732\left(\frac{x}{a}\right)^3 + 0.292\left(\frac{x}{a}\right)^5 - 0.0183\left(\frac{x}{a}\right)^7 + 0.000043\left(\frac{x}{a}\right)^9 - 0.000115\left(\frac{x}{a}\right)^{11} + \cdots\right]$$

$$\text{or } \sigma_0 = \frac{1}{2}\rho U_\infty^2 R^{-\frac{1}{2}}\left(\frac{x}{a}\right)\left[6.973 - 2.732\left(\frac{x}{a}\right)^2 + 0.292\left(\frac{x}{a}\right)^4 - \cdots\right]$$

The position of the point of separation of the B.L. can be found on the condition that the shearing stress must vanish there.

i.e. $\sigma_0 = 0$

i.e. $6.973 - 2.732X + 0.292X^2 - 0.0183X^3 - 0.000043X^4 - 0.000115X^5 = 0.$

where $X = \left(\frac{x}{a}\right)^2$

Solving these it is found that the separation ~~across~~ occurs at $\phi_s = 108.8^\circ$

If the power series were terminated at x^9, the point of separation would turn out to be at $\phi = 109.6^\circ$.

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