$f(x) = x^3 - x - 4$

$f(x_5) = (1.796875)^3 - 1.796875 - 4$

$\boxed{f(x_5) = 3.00 \times 10^{-3} = +ve}$ +ve

7th iteration → Root lies b/w $x_5 = 1.796875$
and $x_4 = 1.78125$

$x_6 = \frac{1.796875 + 1.78125}{2}$

$\boxed{x_6 = 1.7890625}$

Put x_6 in $f(x)$

$f(x) = x^3 - x - 4$

$f(x_6) = (1.7890625)^3 - 1.7890625 - 4$

$\boxed{f(x_6) = -0.0672}$ -ve

8th iteration → Root lies b/w $x_6 = 1.7890625$
& $x_5 = 1.796875$

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$x_7 = \frac{1.7890625 + 1.796875}{2}$

$\boxed{x_7 = 1.79296}$

Put x_7 in $f(x)$

$\boxed{f(1.79296) = -0.02913}$ −ve

th iteration → Root lies b/w $x_7 = 1.79296$ & $x_5 = 1.796875$

$x_8 = \frac{1.79296 + 1.796875}{2}$

$\boxed{x_8 = 1.79491}$

Put x_8 in $f(x)$

$\boxed{f(x_8) = -0.02513}$ −ve

10th iterat^n → Root lies b/w $x_8 = 1.79491$ & $x_5 = 1.796875$

$x_9 = \frac{1.79491 + 1.796875}{2}$

$\boxed{x_9 = 1.79589}$

Put x_9 in $f(x)$

$\boxed{f(x_9) = -0.00375}$ −ve

11th iteration → Root lies b/w $x_9 = 1.79589$ & $x_5 = 1.796875$

$x_{10} = \frac{1.79589 + 1.796875}{2}$

$\boxed{x_{10} = 1.79638}$

final root 1.796

$f(x_2) = (1.875)^3 - 1.875 - 4$

$\boxed{f(x_2) = 0.71679}$ +ve

4th iteration → Root lies between $x_2 = 1.875$ and $x_1 = 1.75$

$$x_3 = \frac{1.875 + 1.75}{2}$$

$\boxed{x_3 = 1.8125}$

Put $x_3 = 1.8125$ in $f(x)$

$f(x) = x^3 - x - 4$

$f(x_2) = (1.8125)^3 - 1.8125 - 4$

$\boxed{f(x_2) = 0.14184}$ +ve

5th iteration → Root lies b/w $x_3 = 1.8125$ & $x_1 = 1.75$

$$x_4 = \frac{1.8125 + 1.75}{2}$$

$\boxed{x_4 = 1.78125}$

Put x_4 in $f(x)$

$f(x) = x^3 - x - 4$

$f(x_4) = (1.78125)^3 - 1.78125 - 4$

$\boxed{f(x_4) = -0.12960}$ −ve

6th iteration → Root lies b/w $x_4 = 1.78125$ & $x_3 = 1.8125$

$$x_5 = \frac{1.78125 + 1.8125}{2}$$

$\boxed{x_5 = 1.796875}$

Put x_5 in $f(x)$

* Bisection (OR) Bolzano method →

Que ① → find a root of the equation $x^3 - x - 4 = 0$ to four places of decimal by bisection method.

Sol^n ① → $\boxed{f(x) = x^3 - x - 4 = 0}$

$\boxed{x=0} \rightarrow f(0) = 0 - 0 - 4 = -4 = -ve$

$\boxed{x=1} \rightarrow f(1) = \cancel{1} - \cancel{1} - 4 = -4 = -ve$

$\boxed{x=2} \rightarrow f(2) = 8 - 2 - 4 = 8 - 6 = 2 = +ve$

1st iteration → Root lies between $x = 1$ & $x = 2$

$x_0 = \frac{1+2}{2} = \frac{3}{2} = 1.5$

$\boxed{x_0 = 1.5}$

Put $x_0 = 1.5$ in $f(x)$

$f(x) = x^3 - x - 4$

$f(x_0) = (1.5)^3 - 1.5 - 4$

$\boxed{f(x_0) = -2.125}$ $-ve$

2nd iteration → Root lies between $x_0 = 1.5$ & $x = 2$

$x_1 = \frac{1.5+2}{2}$

$\boxed{x_1 = 1.75}$

Put $x_1 = 1.75$ in $f(x)$

$f(x) = x^3 - x - 4$

$f(x_1) = (1.75)^3 - 1.75 - 4$

$\boxed{f(x_1) = -0.390625}$ $-ve$

3rd iteration → Root lies between $x_1 = 1.75$ and $x = 2$

$x_2 = \frac{1.75+2}{2}$

$\boxed{x_2 = \frac{3.75}{2} = 1.875}$

Put x_2 in $f(x)$

$f(x) = x^3 - x - 4$

$\because f(2.625) = -ve$ and $f(2.75) = +ve$, root lies betn 2.625 & 2.75.

Fourth Approx. $x_4 = \dfrac{\overset{(-)}{2.625} + \overset{(+)}{2.75}}{2} = 2.6875$ Ans

$f(2.6875) = -0.3391$

$x_5 = 2.71875$

$x_6 = 2.7031$

$x_7 = 2.7109$

$x_8 = 2.707$

$x_9 = 2.7051$

$x_{10} = 2.7061$

$x_{11} = 2.7066$

$x_{12} = 2.7064$

$x_{13} = 2.7065$

$x_{14} = 2.7065$

$\therefore x_{13} = x_{14} = 2.7065$

Hence the root is 2.7065.

Problem 1. Find a root of the equation $x^3 - 4x - 9 = 0$ using the bisection method

(i) in four stages

(ii) correct to four decimal places

Let $f(x) = x^3 - 4x - 9$

$f(0) = -9$

$f(1) = -12$

$f(2) = -9$

$f(3) = +6$

$\because f(2) = -ve$ and $f(3) = +ve$

so the root lies bet.ⁿ 2 & 3.

First Approx.

$x_1 = \frac{2+3}{2} = 2.5$

$f(2.5) = -3.375$

$\because f(2.5) = -ve$ and $f(3) = +ve$,

so the root lies bet.ⁿ 2.5 and 3.

Second Approx.

$x_2 = \frac{\overset{(-)}{2.5} + \overset{(+)}{3}}{2} = \frac{5.5}{2} = 2.75$

$f(2.75) = 0.797 = +ve$

$\because f(2.5) = -ve$ and $f(2.75) = +ve$

so the root lies bet.ⁿ 2.5 & 2.75.

Third Approx.

$x_3 = \frac{\overset{(-)}{2.5} + \overset{(+)}{2.75}}{2} = 2.625$

$f(2.625) = -1.412 = -ve$

Working Rule -

① Find a_0, b_0 s.that $f(a_0) \cdot f(b_0) < 0$

② $m_1 = \frac{a_0+b_0}{2}$

If $f(a_0) f(m_1) < 0$

$I_1 = (a_1, b_1)$

a_0, m_1

$m_2 = \frac{a_1+b_1}{2}$

If $f(a_0) f(m_1) > 0$

$I_1 = (a_1, b_1)$

$m_1 \quad b_0$

$m_2 = \frac{a_1+b_1}{2}$

In general, $m_{k+1} = \frac{a_k+b_k}{2}$, $k = 0, 1, 2 \ldots$

$$(a_{k+1}, b_{k+1}) = \begin{cases} (a_k, m_{k+1}) & \text{if } f(a_k) \cdot f(m_{k+1}) < 0 \\ (m_{k+1}, b_k) & \text{if } f(a_k)\, f(m_{k+1}) > 0 \end{cases}$$

Q1 Perform four iterations of the bisection method to obtain the smallest positive root of the equation

$$f(x) = x^3 - 5x + 1 = 0$$

- $f(a)\cdot f(b)<0$
- a root of $f(x)=0$ lies in the interval (a,b)

$$I_0=(a_0,b_0) \quad a_0=a,\ b_0=b$$

- Bisect I_0 at the point $m_1=\frac{a_0+b_0}{2}$

If $f(a_0)\cdot f(m_1)<0$

Take $I_1=(a_0,m_1)$

$I_1=(a_1,b_1)$

where $a_1=a_0,\ b_1=m_1$

$m_2=\frac{a_1+b_1}{2}$

If $f(m_1)f(b_0)<0$

Take $I_1=(m_1,b_0)$

$I_1=(a_1,b_1)$

$)<0$

$b_1)$

$I_0\supset I_1\supset I_2\supset I_3$ -----

- After repeating this bisection process q times we either find the root or find the interval I_q of length $\frac{(b_0-a_0)}{2^q}$ which contains the root.

- When to stop -

① Accuracy upto 4 decimal places

.32915

.32916

② No. of iterations are given

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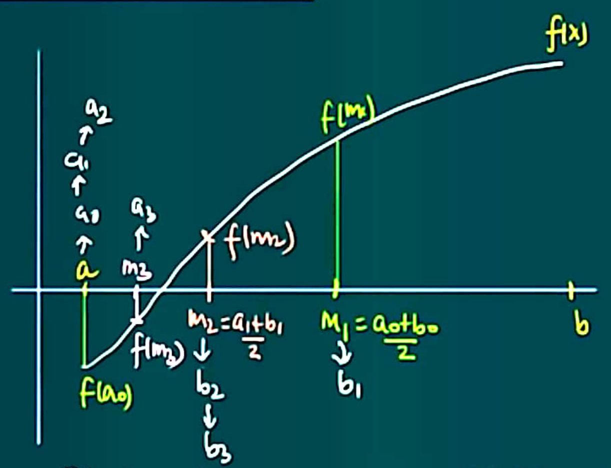
The Bisection Method



If $f(a_1)\,f(m_2) < 0$

(a_1, m_2)

$I_2 = (a_2, b_2)$

$f(m_3)\cdot f(b_2) < 0$

If $f(m_2)\,f(b_1) < 0$

(m_2, b_1)

- $f(a)\cdot f(b) < 0$
- a root of $f(x)=0$ lies in the interval (a,b)

$I_0 = (a_0, b_0)$ $\quad a_0 = a,\ b_0 = b$

- Bisect I_0 at the point $m_1 = \frac{a_0+b_0}{2}$

If $f(a_0)\cdot f(m_1) < 0$

Take $I_1 = (a_0, m_1)$

$I_1 = (a_1, b_1)$

where $a_1 = a_0,\ b_1 = m_1$

$m_2 = \frac{a_1+b_1}{2}$

If $f(m_1)\,f(b_0) < 0$

Take $I_1 = (m_1, b_0)$

$I_1 = (a_1, b_1)$

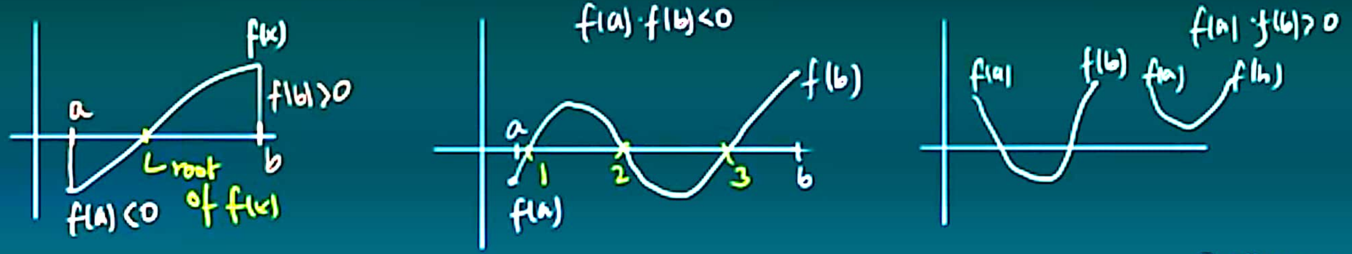
$I_0 \supset I_1 \supset I_2 \supset I_3$

THE BISECTION METHOD

This method is based on the repeated application of the INTERMEDIATE Value Theorem.

Intermediate Value Theorem -

If $f(x)$ is a cts function on some interval $[a,b]$ and $f(a) \cdot f(b) < 0$, then the equation $f(x) = 0$ has atleast one real root or an odd number of real roots in the interval $[a,b]$.



$\because f(0.5) = +ve$ and $f(1) = -ve$, the root lies b/w 0.5 & 1

$$x_2 = \frac{0.5+1}{2} = 0.75$$

$$f(x_2) = f(0.75) = \cos(57.3 \times 0.75) - (0.75)e^{0.75} = -0.8561 = -ve$$

$\because f(0.5) = +ve$ & $f(0.75) = -ve$, the root lies b/w 0.5 & 0.75.

$$x_3 = \frac{0.5+0.75}{2} = 0.625$$

$$f(x_3) = f(0.625) = -0.3567 = -ve$$

$\because f(0.5) = +ve$ & $f(0.625) = -ve$, the root lies b/w 0.5 and 0.625

$$x_4 = \frac{0.5+0.625}{2} = 0.5625$$

Hence, the root is 0.5625 Ans

$\because f(0) = +ve$ and $f(1) = -ve$, the root lies b/w 0 & 1.

$$x_1 = \frac{0+1}{2} = 0.5\,;$$

$$f(x_1) = f(0.5) = \cos(57.3 \times 0.5) - (0.5)e^{0.5} = 0.0532 = +ve$$

$\because f(0.5) = +ve$ and $f(1) = -ve$, the root lies b/w 0.5 & 1.

$$x_2 = \frac{0.5+1}{2} = 0.75$$

$$f(x_2) = f(0.75) = \cos(57.3 \times 0.75) - (0.75)e^{0.75} = -0.8561 = -ve$$

$\because f(0.5) = +ve$ & $f(0.75) = -ve$, the root lies b/w 0.5 & 0.75.

$$x_3 = \frac{0.5+0.75}{2} = 0.625$$

$$f(x_3) = f(0.625) = -0.3567 = -ve$$

$\because f(0.5) = +ve$ & $f(0.625) = -ve$, the root lies b/w 0.5 and 0.625

$$x_4 = \frac{0.5+0.625}{2} = 0.5625$$

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Que ① Find a positive root of $\cos x - x e^{x} = 0$ in four steps.

$\cos x = x e^{x}$

$\log_{10}$

Solⁿ Let $f(x) = \cos x - x e^{x}$

$f(0) = \cos(57.3 \times 0) - 0.e^{0} = 1 = +ve$

$f(1) = \cos(57.3 \times 1) - 1.e^{1} = -2.18 = -ve$

$\because f(0) = +ve$ and $f(1) = -ve$, the root lies b/w 0 & 1.

$$x_1 = \frac{0+1}{2} = 0.5;$$

$f(x_1) = f(0.5) = \cos(57.3 \times 0.5) - (0.5)e^{0.5} = 0.0532 = +ve$

$\because f(0.5) = +ve$ and $f(1) = -ve$, the root lies b/w 0.5 & 1.

$$x_2 = \frac{0.5+1}{2} = 0.75$$

$f(x_2) = f(0.75) = \cos(57.3 \times 0.75) - (0.75)e^{0.75} = -0.8561 = -ve$

$\because f(0.5) = +ve$ & $f(0.75) = -ve$, the root lies b/w 0.5 & 0.75.