De Broglie wavelength and matter waves

Lecture 4

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The wave nature of matter:

In 1905 Albert Einstein introduced the quantum theory of radiation to explain photoelectric effect and he introduced the new concept that light radiation is consisted of some small particles called photons.

With the introduction of this new theory Physicists were obliged to admit a duel nature of radiation, wave and particle. Albert Einstein Proposed that energy and mass are interconnected as a function of motion such that,

$$E = mc^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

This is called as Matter-Energy duality.

In 1924, French physicists Louis de Broglie suggested that like radiation matter has dual nature and derived an expression for wavelength of matter wave.

The mathematical expression for wavelength of wave associated with an electron is

$$\lambda = h/p \quad \text{...........................(2)}$$

Where λ wavelength, h Planck's constant and p momentum of the particle.

De Broglie derived the wave expression using the general equation of a standing wave system and principle of relativity.

Consider the case of an electron, as a standing wave system in the region of space occupied by the particle. At an instant $t_\circ$, at a point ($x_\circ$, $y_\circ$, $z_\circ$) the periodic change for the electron wave can be written as

$$\varphi = \varphi_\circ \sin 2\pi \nu_\circ t_\circ \quad \text{...................(3)}$$

Where $\varphi_\circ$ amplitude at this point and $\nu_\circ$ is the frequency of the matter wave as observed by an observer at rest with respect to the particle.

Now applying relativistic transformation equation for t we have

$$t_\circ = \frac{t - vx/c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore \varphi = \varphi_\circ \sin 2\pi\nu_\circ \frac{t - vx/c^2}{\sqrt{(1-v^2/c^2)}} \quad \text{.................(4)}$$

The standard wave equation is

$$y = A \sin \frac{2\pi}{T}(t - {}^{x}/_{u}) \quad \text{..........................(5)}$$

Where A is the amplitude, T time period and u velocity of the wave along x axis.

Comparing equation (4) and (5) we get

$$u = c^2/v \quad \text{.......................................(6)}$$

and $$\frac{1}{T} = \frac{\nu_\circ}{\sqrt{(1-v^2/c^2)}} = \nu \quad \text{..........................(7)}$$

From Einstein famous mass energy relation we have

$$E = m_\circ c^2 = h\nu_\circ$$

$$\nu_\circ = \frac{m_\circ c^2}{h}$$

$$\nu = \frac{m_\circ c^2}{h\sqrt{(1-\frac{v^2}{c^2})}}$$

Since m = $\frac{m_\circ}{\sqrt{(1-\frac{v^2}{c^2})}}$ therefore $\nu = \frac{mc^2}{h}$ …………….(8)

Now from equation (6) and (8) we have

$$\lambda = \frac{velocity}{frequency}$$

$$= \frac{u}{\nu} = \frac{c^2/v}{mc^2/h} = \frac{h}{mv}$$

$$\lambda = \frac{h}{p} \quad \text{………………………………….(9)}$$

Equation (9) is the expression for de Broglie wavelength for matter wave which is the ratio of Planck's constant h to the momentum mv of the particle.

Any queries ?

Thank You