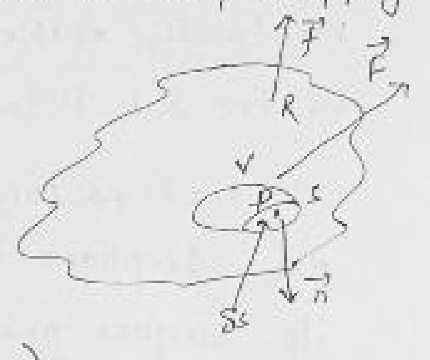
§: (2.1) Body force and external force:

Those forces which act on all elements of volume of a fluid is called a body force or external force. It is measured per unit mass in general.

Those force which act on surface elements on the bounding surface or arbitrary internal surface within the given fluid medium are called surface force. It is measurs per unit area on the surface.

2.2. Stress force or stress at a point:

It is a risistance force within the fluid during action of external forces. Suppose we consider a fluid medium occupying a region R. At different point on it external forces $\vec{F}$ and surface force $\vec{f}$ are acting of three forces will be transmitted from one portion of the medium to the other portion.



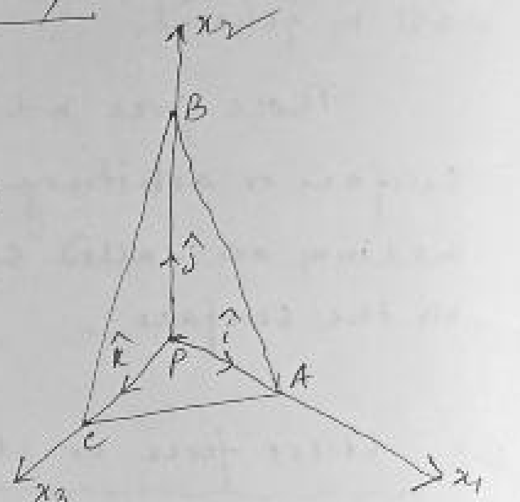
If we consider a volume v of fluid enclosed by S within R, then there will be interaction of the fluid inside S with the fluid outside S. Hence there will be a surface force at different point on S in addition to the external forces at different point on it.

Let us consider a small elementary area ds at P on S and $\vec{n}$ be the outward unit normal to ds. Let $\delta\vec{f}$ be the resultant force exerted accross ds upon the material outside V. Avarage force per unit area on ds is given by $\frac{d\vec{f}}{ds}$. Cauchy stress principle states that $\frac{d\vec{f}}{ds}$ tends to definite limit force as

$ds \to 0$. This resulting force or vector is called stress vector or stress force at the point P.

Stress is sometimes called the traction vector also.

2.4. Stress vector interms of stress matrix:



Generally, stress vector depends on the position P and on the normal $\vec{n}$ at P on surface S. (Stress is defined in a connection with the surface always). Through $\vec{n}$ does denotes many directions at the point P with different ds, yet together the stress vector $t^{(\vec{n})}$ at P.

We donot require all the directions. If stress vector on three mutually orthogonal planes at P can be defined then, stress vector at P can be defined in general terms.

∴ Let us consider a small tetrahedron PABC of sides dx_1, dx_2, dx_3. Suppose it is in equilibrium under the stress force on its surface and external (body) force within it.

Now, external forces are proportional to the cube of small distances. Hence, for equilibrium of the tetrahedron surface forces must be in equilibrium. Let $\vec{n}$ be the unit normal on ABC and $\hat{i}$ on PCB, $\hat{j}$ on APC, $\hat{k}$ on APB. Hence the stress forces on the tetrahedron are $t_s^{(\hat{n})}, t_s^{(\hat{i})}, t_s^{(\hat{j})}, t_s^{(\hat{k})}$ where s is equal to 1, 2, 3 and it gives three components each along $\vec{Px_1}, \vec{Px_2}, \vec{Px_3}$ direction.

Total force on PAB is $t_s^{(\hat{k})} ds_3$

total force on PBC is $t_s^{(\hat{i})} ds_1$

and total stress force on PCA is $t_s^{(\hat{j})} ds_2$.

and total stress force on ABC is $t_s^{(\hat{n})} ds$.

Here,

ds = Area of ABC

ds_1 = Area of PBC

ds_2 = Area of PCA

Since the stress are in equilibrium, therefore,

$$ds \cdot t_s^{(\hat{n})} = ds_1\, t_s^{(\hat{i})} + ds_2\, t_s^{(\hat{j})} + ds_3\, t_s^{(\hat{k})}$$

$$\Rightarrow t_s^{(\hat{n})} = \frac{ds_1}{ds} t_s^{(\hat{i})} + \frac{ds_2}{ds} t_s^{(\hat{j})} + \frac{ds_3}{ds} t_s^{(\hat{k})}$$

$$= n_1\, t_s^{(\hat{i})} + n_2\, t_s^{(\hat{j})} + n_3\, t_s^{(\hat{k})}$$

where $\vec{n} = (n_1, n_2, n_3)$, n_1, n_2, n_3 give the direction cosines of the normal to ABC.

$$\left[\begin{array}{l} \text{i.e.} \quad ds \cdot n_1 = ds \cos\alpha = ds_1 \\ \qquad ds \cdot n_2 = ds \cos\beta = ds_2 \\ \qquad ds \cdot n_3 = ds \cdot \cos\gamma = ds_3 \end{array}\right]$$

If the components of $t_s^{(\hat{i})}$ be $(\sigma_{11}, \sigma_{12}, \sigma_{13})$

$t_s^{(\hat{j})}$ be $(\sigma_{21}, \sigma_{22}, \sigma_{23})$

$t_s^{(\hat{k})}$ be $(\sigma_{31}, \sigma_{32}, \sigma_{33})$

along the coordinates axes, then

$$t_1^{(\hat{n})} = n_1 \sigma_{11} + n_2 \sigma_{21} + n_3 \sigma_{31}$$

$$t_2^{(\hat{n})} = n_1 \sigma_{12} + n_2 \sigma_{22} + n_3 \sigma_{32}$$

$$t_3^{(\hat{n})} = n_1 \sigma_{13} + n_2 \sigma_{23} + n_3 \sigma_{33}$$

These relations can be written as

$$\left[t_1^{(\hat{n})}, t_2^{(\hat{n})}, t_3^{(\hat{n})}\right] = \left[n_1\ n_2\ n_3\right]\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

$$\therefore \left[t_j^{(\hat{n})}\right] = [n_j][\sigma_{ij}] = \left[\sigma_{ji} n_j\right]$$

This matrix $[\sigma_{ji}]$ is called stress matrix at a point P. If $\vec{n}$ or n_j is given $t_j^{(\hat{n})}$ can be calculated. Here $\sigma_{11}, \sigma_{22}, \sigma_{33}$ are called normal stresses as they act normal to the coordinates planes, other components are called shearing stresses.

A stress component is positive if it acts in the +ve direction of the coordinate axes and acts on a plane where outward normal in the positive direction of the coordinate plane.

Stress motion is symmetrical in nature

i.e. $\sigma_{ij} = \sigma_{ji}$.

If the fluid be at rest, the stress vector $t_s^{(\vec{n})}$ on an arbitrary surface element is collinear with the normal $\hat{n}$ of the surface. In this case

$$\sigma_{11} = \sigma_{22} = \sigma_{33} = -p.$$

and $\sigma_{12} = \sigma_{23} = \sigma_{31} = 0$.

p is hydrostatic pressure and $-$ve sign gives the nature of the force.

$$\sigma_{ij} = -p\delta_{ij}$$

For a fluid in motion the shearing stress components are usually not zero and it is resolved into the form

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij}$$

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where τ_{ij} is called viscous stress components. All real fluids are compressible and viscous. However, the characteristic of the fluid very widely in different cases.

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