Alternating Current
Lecture 8

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Resonance Frequency of series LCR Circuit:

Now if the two reactance is same ie. $X_L = X_C$, then the angular frequency at which this occurs is called the resonant frequency and produces the effect Resonance. The magnitude of the current depends upon the frequency. When the current is maximum, then impedance is minimum and vice versa. Again the phase angles are as

$$\cos\emptyset = \frac{R}{Z} \rightarrow (xvi)$$

$$sin\emptyset = \frac{X_L - XC}{Z} \rightarrow (xvii)$$

$$tan\emptyset = \frac{X_L - XC}{R} \rightarrow (xviii)$$

In $LCR\ series$ circuit resonance occurs when the value of inductive and capacitive reactance have equal magnitude but have a phase difference of 180^0. Thus they cancel each other. This is known as resonance frequency of series $LCR\ circuit$.

$$X_L = X_C \rightarrow (i)$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \; Hz \rightarrow (ii)$$

$$\omega_r = \frac{1}{\sqrt{LC}} \; radian \rightarrow (iii)$$

This makes the series LC combination act as a short circuit with the only opposition to current flow in series resonance circuit being the resistance R.

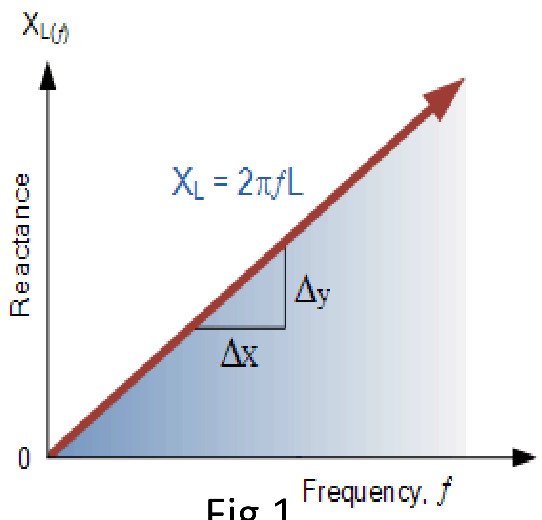

Fig 1

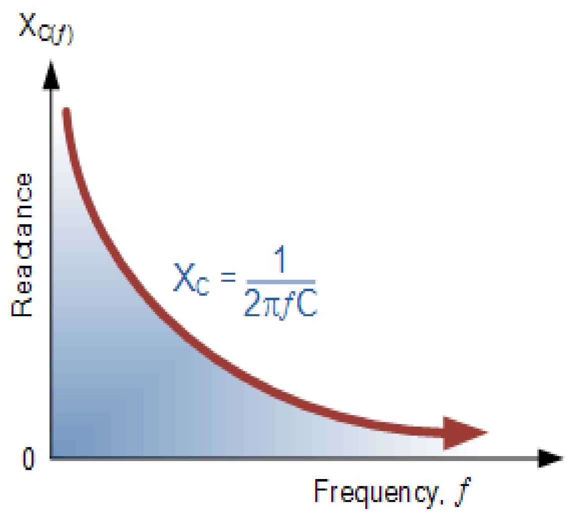

Fig 2

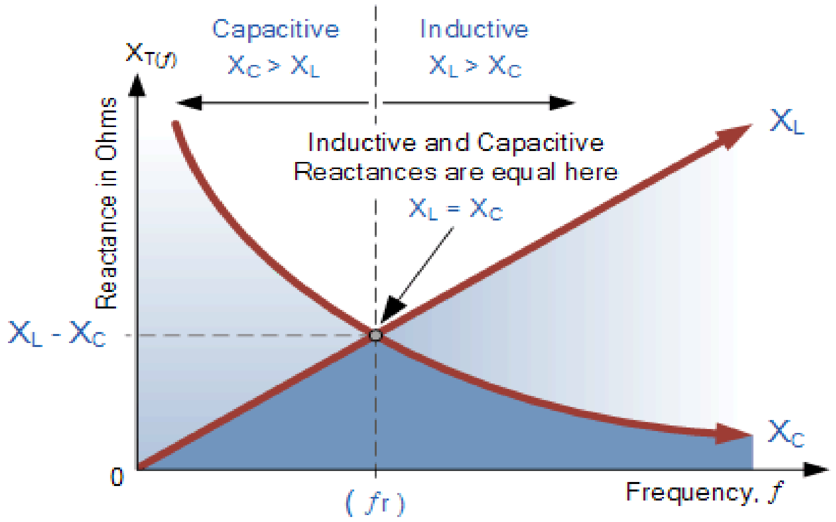

Fig 3

In complex form the resonant frequency is the frequency at which the total impedance of the series LCR circuit becomes purely real i.e. no imaginary part exist. So total impedance is equal to resistance only i.e. $Z = R$. The frequency response curve of a series resonance circuit shows that the magnitude of current is a function of frequency and if we plot the graph found that response starts at zero reaches maximum value at the resonance frequency when $I_{max} = I_R$ and drops again to nearly zero as f becomes infinite.

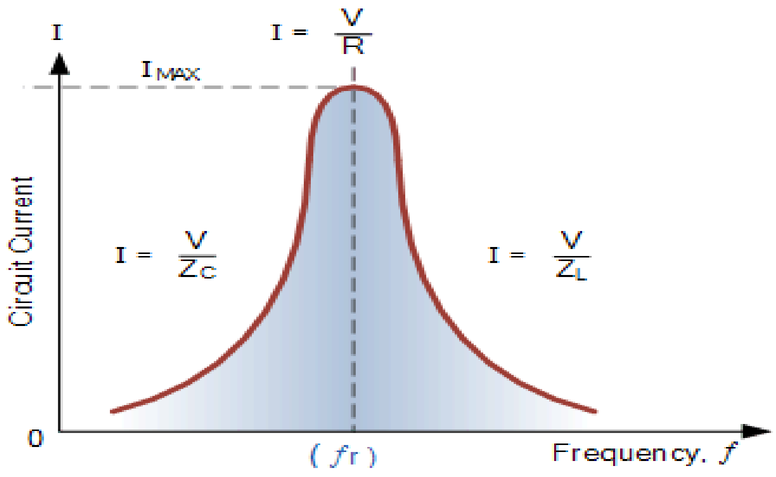

Fig 4

The result is that the magnitude of the voltage across the inductor (L) and capacitor (C) can becomes many times larger than the supply voltage even at resonance but they are equal and opposite and cancel each other. The series resonance circuit only function on resonant frequency and known as $Acceptor\ Circuit$ because at resonance the impedance of the circuit is minimum so easily accept the current whose frequency is equal to resonant frequency. At this frequency the source voltage and current are must be at phase each other. Then the phase angle between the voltage and current of a series resonance circuit is also a function of frequency for a fixed supply voltage and which is zero at the resonant frequency point.

If a series resonance circuit is driven by variable frequency at constant source then the magnitude of current is proportional to impedance. Therefore the power absorbed by the circuit must be maximum value as

$$P = I^2 Z \rightarrow (iv)$$

Now if the frequency increase or decrease until the average power absorbed by the resistance in series circuit is half that of its maximum value at resonance we produce two frequency points called half power point which are $3\ dB$ down from the maximum, taking $0\ dB$ as the maximum current reference. These $3\ dB$ points gives a current value of 70.7% of its maximum resonant value which is defined as

$$0.5(I^2 R) = (0.707I)^2 R \rightarrow (v)$$

Then point corresponding to the lower frequency at half power is called the lower cut-off frequency (f_L) and the point corresponding to upper frequency at half power point being called the upper cut-off frequency (f_H). The distance between the two points is called the Bandwidth and represented by ($f_H - f_L$). This is the range of frequencies over which at least half of the maximum power and current is provided as shown in Fig 5.

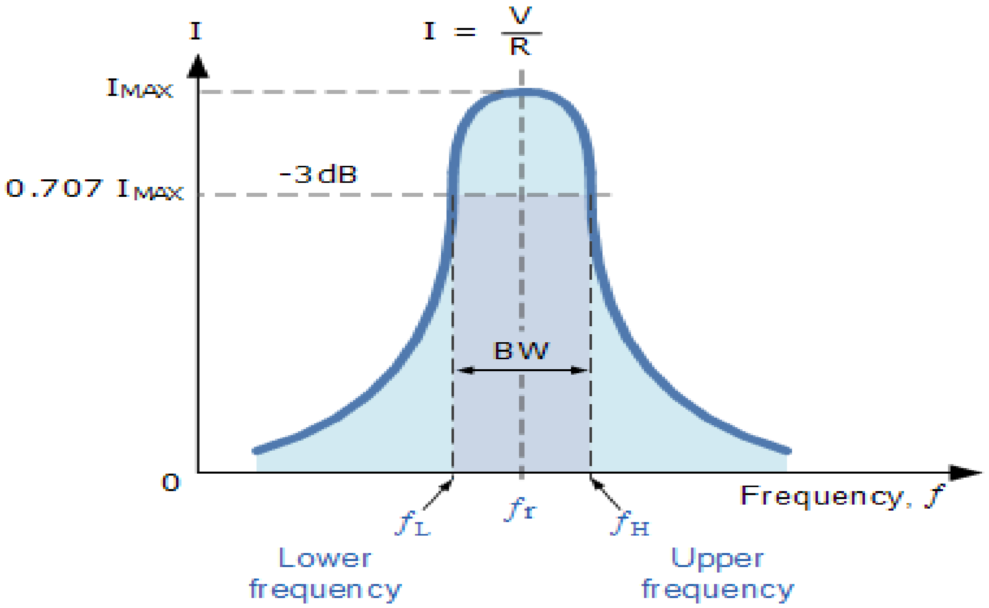

Fig 5

Again the frequency response of current magnitude relates to the sharpness of resonance in the series resonance circuit. The sharpness of the peak is measured quantitively and called the Quality Factor (Q) of the circuit. The quality factor relates the peak energy stored in the circuit to the energy dissipated during each cycle of oscillation. This means that it is the ratio of resonant frequency to bandwidth and higher the circuit Q the smaller the bandwidth.

$$Q = \frac{f_r}{B.W.}$$

$$Q = \frac{f_r}{f_H - f_L} \rightarrow (vi)$$

As bandwidth is taken between $3dB$ points, the selectivity of the circuit is the measure of its ability to reject any frequencies either side of these points. A more selective circuit will have narrow bandwidth and less selective circuit have wider bandwidth. The selectivity of series resonance can be controlled by adjusting the resistance only by keeping the other two component are same.

$$Q = \frac{X_L}{R} = \frac{\omega_r L}{R} = \frac{X_C}{R} = \frac{1}{\omega_r RC} = \frac{1}{R}\sqrt{\frac{L}{C}} \rightarrow (vii)$$

Again at resonant frequency(ω_r)

$$Z = Minimum, I_S = Maximum$$

Then

$$I_{max} = \frac{V_{max}}{Z} = \frac{R^2}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \rightarrow (viii)$$

Again

$$\omega_H = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \rightarrow (ix)$$

And

$$\omega_L = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \rightarrow (x)$$

Parallel LCR Circuit:

In case of parallel LCR circuit the supply voltage V_S is common to all three components while supply current has three parts; I_R , I_L , I_C are current passing through resistance, inductance and capacitance respectively. But the current flowing through each component are different to each other and also to supply current I_S .

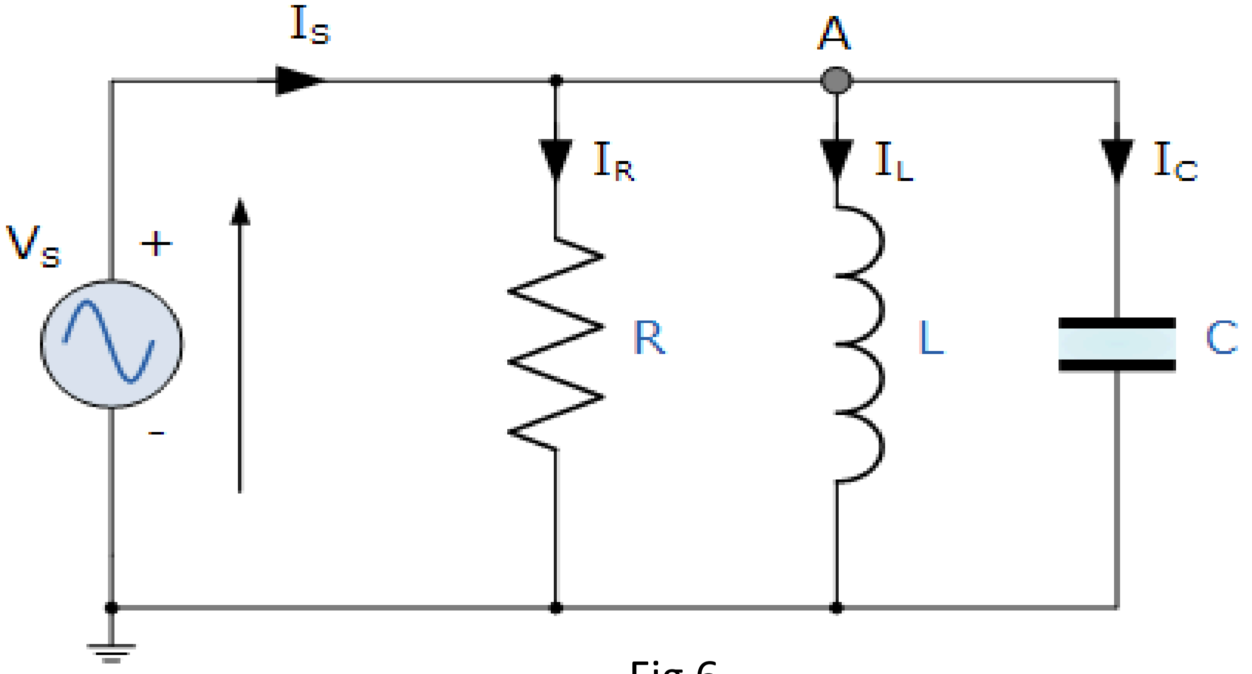

Fig 6

The total current drawn from the supply will not be mathematical sum of three individual current but their vector sum.

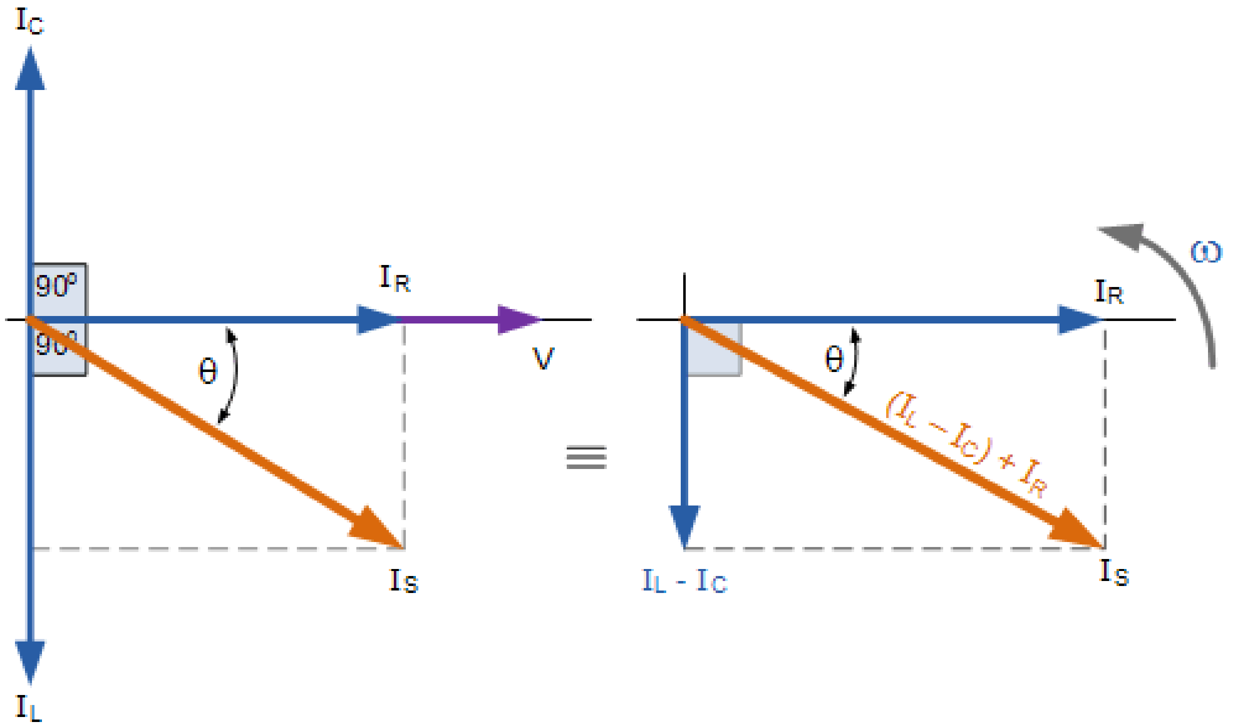

Fig 7

Let us consider the parallel LCR circuit with AC source, then

$$V_S = V_0 sin\omega t \rightarrow (xi)$$

$$I_S = I_R + I_C + I_L \rightarrow (xii)$$

Now

$$I_R = \frac{V_S}{R} = \frac{V_O sin\omega t}{R} \rightarrow (xiii)$$

$$I_C = \frac{dQ}{dt} = \frac{d(CV_S)}{dt} = \frac{d(CV_0 sin\omega t)}{dt} = CV_0(cos\omega t\omega) = C\omega V_0 cos\omega t \rightarrow (xiv)$$

$$I_L = \int dI_L = \int \frac{V_0 sin\omega t}{L} dt = \frac{V_0}{L}\left(\frac{cos\omega t}{\omega}\right) = -\frac{V_0}{L\omega} cos\omega t \rightarrow (xv)$$

Therefore

$$I_S = \frac{v_0 sin\omega t}{R} + C\omega V_0 cos\omega t - \frac{V_0}{L\omega} cos\omega t \rightarrow (xvi)$$

$$I_S = V_0 \left[\frac{sin\omega t}{R} + \left(C\omega - \frac{1}{L\omega}\right) cos\omega t\right] \rightarrow (xvii)$$

Putting $\frac{1}{R} = Acos\theta$ and $\left(C\omega - \frac{1}{C\omega}\right) = A\ sin\theta$ we can write

$$I_S = V_0[Asin\omega tcos\theta + Acos\omega tsin\theta]$$

$$I_S = V_0 Asin(\omega t + \theta)$$

$$I_S = I_0 sin(\omega t + \theta) \rightarrow (xviii)$$

Where $I_0 = AV_0$ is the peak value of current. Again

$$\frac{Asin\theta}{Acos\theta} = \frac{\left(C\omega - \frac{1}{L\omega}\right)}{\frac{1}{R}} = R\left(C\omega - \frac{1}{L\omega}\right)$$

$$tan\theta = R\left(C\omega - \frac{1}{L\omega}\right) \rightarrow (xix)$$

Resonance in parallel circuit occurs when I_0 is minimum which happens for minimum value of A. Now

$$A^2cos^2\theta + A^2sin^2\theta = \frac{1}{R^2} + \left(C\omega - \frac{1}{L\omega}\right)^2$$

$$A^2 = \frac{1}{R^2} + \left(C\omega - \frac{1}{L\omega}\right)^2 \rightarrow (xx)$$

$$A = \sqrt{\frac{1}{R^2} + \left(C\omega - \frac{1}{L\omega}\right)^2} \rightarrow (xxi)$$

It is called Admittance of the LCR circuit and related to Impedance as follows

$$Z = \frac{1}{A} = \frac{1}{\sqrt{\frac{1}{R^2} + \left(C\omega - \frac{1}{L\omega}\right)^2}} \rightarrow (xxii)$$

Peak value of the current is

$$I_0 = AV_0 = V_0\sqrt{\frac{1}{R^2} + \left(C\omega - \frac{1}{L\omega}\right)^2} \rightarrow (xxiii)$$

Now $A = A_{min}$, if

$$C\omega - \frac{1}{L\omega} = 0$$

$$\omega^2 = \frac{1}{L\omega}$$

$$\omega = \frac{1}{\sqrt{LC}} \rightarrow (xxiv)$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \rightarrow (xxv)$$

At resonance

$$tan\theta = R\left(C\omega - \frac{1}{L\omega}\right) = 0$$

$$\theta = 0 \rightarrow (xxvi)$$

Thus current and voltage are in phase at resonance. Thus frequency of alternating voltage becomes equal to natural frequency of oscillation of circuit and the resonance occure and frequency is called parallel resonance frequency. At this frequency of parallel resonance circuit, does not allow any current to flow through it and impedance of the parallel resonance circuit is maximum at this frequency.

This circuit reject current corresponding to parallel resonance frequency and allow other frequency to pass through it. Therefore this circuit is called $Rejector\ Circuit$.

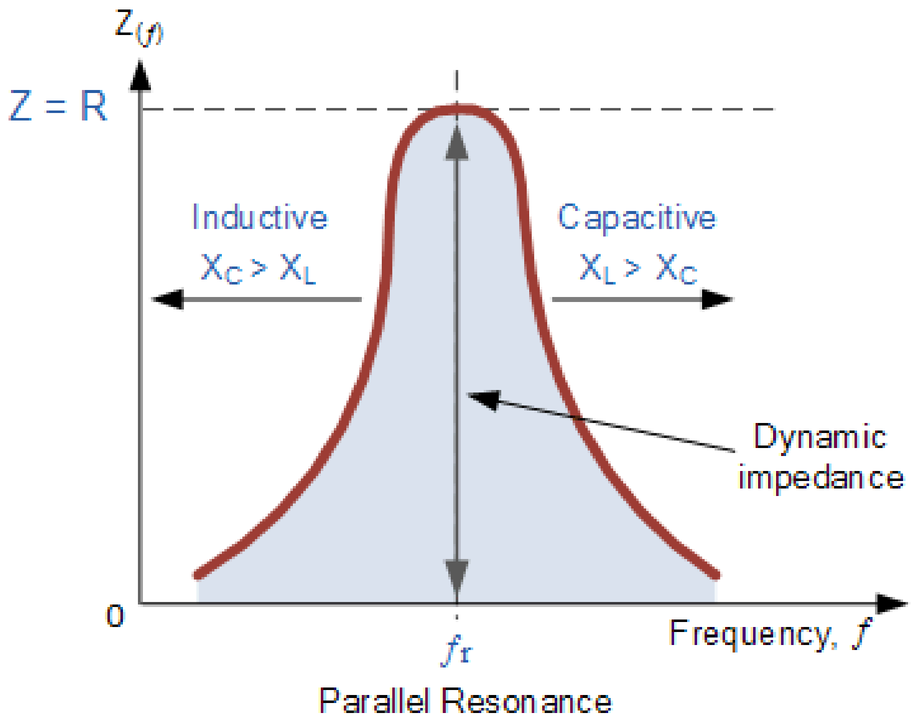

Fig 8

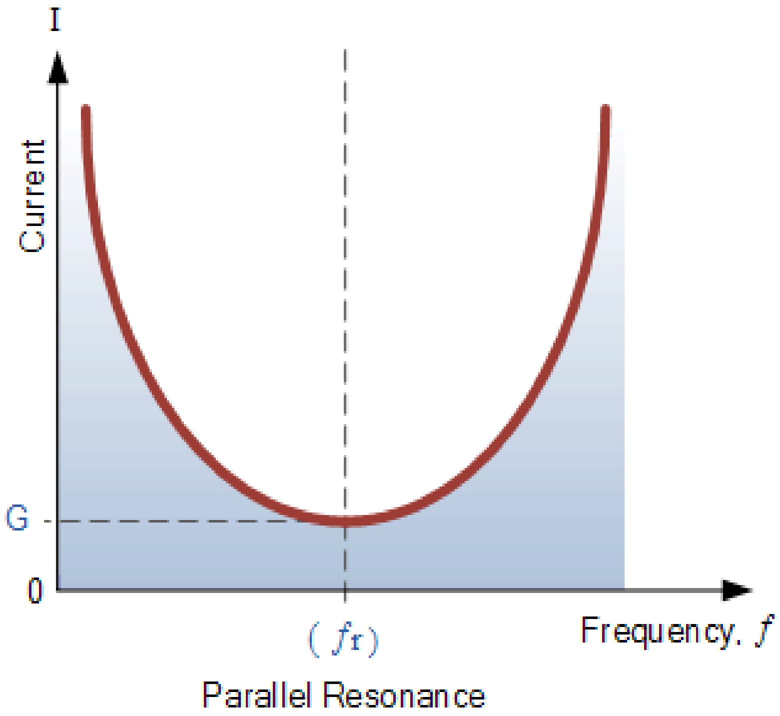

Fig 9

If the parallel circuit impedance is at its maximum at resonance the consequently the circuit’s admittance must be at minimum and one of the characteristic of a parallel resonance circuit is that admittance is very low limiting the current. Again unlike the series resonance circuit the resistance in the parallel resonance circuit has a damping effect on the circuit bandwidth making the circuit less selective. Again at the resonance frequency the admittance of the circuit is equal to conductance (G) given by $\frac{1}{R}$, because in a parallel resonance circuit the imaginary part of admittance i.e. the susceptance is zero. Again at resonance frequency the current drawn from the supply must be in phase with the applied voltage as effectively there is only the resistance present in the parallel circuit so the power factor becomes unity i.e. $\theta = 0$. Again at resonance the current flowing through the circuit must also be as its minimum.

At resonance frequency the inductive and capacitive current are 180^0 out of phase. Since the current flowing through the parallel resonance circuit is the product of voltage divided by impedance is at maximum value. Therefore the current at this frequency wii be at its minimum value of $\frac{V}{Z} = \frac{V}{R} \, (Z = R)$ and the graph of current against frequency is shown in Fig 7. The frequency response curve of parallel resonance circuit shows that the magnitude of current is a function of frequency and graph shows that the response starts at its maximum value, reaches its minimum value at the resonance frequency when $I_{min} = I_R$ and the increases again to maximum as f becomes infinite. The result of this is that the magnitude of the current flowing through the inductor and capacitor, the tank circuit becomes many times larger than the supply current.

Even at resonance as they are equal and opposite and therefore they cancel each other. As a parallel resonance circuit. It is known as rejector circuit because at resonance the impedance of the circuit is at its maximum thereby suppressing or rejecting the currents whose frequency is equal to its resonant frequency. It is also known as current resonance. The bandwidth of parallel resonance circuit is defining exactly the same way as for the series resonance circuit. The upper and lower cut off frequency are given by f_H and f_L respectively denote the half power frequencies where the power dissipated in the circuit is half of the full power dissipated at the resonant frequency $0.5(IR^2)$ which gives the same $3dB$ points at the current value that is equal to 70.7% of its maximum resonant value $(0.707I)^2R$

As with the series circuit if the resonant frequency remains constant, an increase in quality factor (Q) will cause a decrease in bandwidth and the other hand an decrease in quality factor will cause in increase in bandwidth. The bandwidth is defined as

$$BW = \frac{f_R}{Q} = f_H - f_L \rightarrow (xxvii)$$

The selectivity or $Q - factor$ of a parallel resonance circuit is generally defined as the ratio of the circulating branch current to the supply current as

$$Q = \frac{R}{2\pi f L} = 2\pi f C R = R\sqrt{\frac{C}{L}} \rightarrow (xxviii)$$

It is noted that the quality factor for parallel resonance circuit is inverse of the expression for the quality factor of series resonance circuit. Also in series resonance circuit the quality factor gives the voltage magnification of the circuit while in a parallel circuit it gives the current magnification of the circuit.

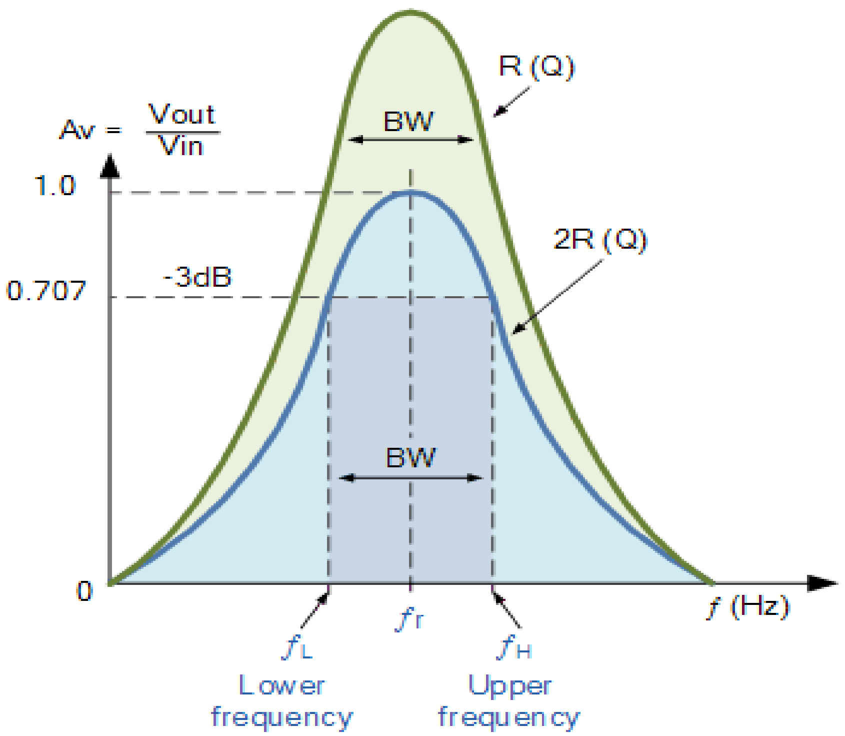

Fig 10