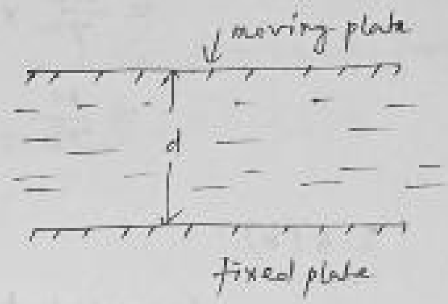
2.6 (a) ~~Physical~~ Physical Significance of viscosity:

Viscosity is a sticking property of a fluid. Viscosity of a substance is measured by the tangential stress (shearing stress) per unit area with which the fluid is in contact. Let us consider two horizontal plates at a distance 'd' apart, the region between the plates is filled up by viscous fluid.



If the lower plate is kept fixed and the upper plate is allowed to move, then the fluid inside begin to move. Motion is due to tangential force along the plate. This forces is measured in terms of constant μ. Here μ is called viscosity constant. If the plate are at a unit distance and the upper plate is moving with unit velocity, then the tangential stress on the plate is represented by μ. This is the physical significance of μ.

If we consider any horizontal plane in the fluid, the portions of fluid above and below it exert on one another a horizontal fraction $\mu \frac{U}{d}$ per unit area, i.e. μ times the velocity gradient. And in more general case, where we donot assume the velocity gradient to be uniform, if z be an axis at right angles to the plane and u be the velocity at any point, the tangential force on either portion into which a horizontal plane through the point divides the fluid is measured by $\mu \frac{du}{dz}$ per unit area.

Dimension of μ:

We have $\mu = \dfrac{\text{Force/unit area}}{\frac{du}{dz}}$; | $\dfrac{du}{dz} = \dfrac{\text{Vel.}}{\text{length}}$

$$\therefore \mu = \frac{\frac{ML}{T^2}}{L^2} \div \frac{L/T}{L}$$

$$= \frac{ML}{T^2} \times \frac{1}{L^2} \times \frac{LT}{L}$$

$$= \frac{M}{TL} = MT^{-1}L^{-1}.$$

2.6 (b):

components of σ_{ij} along the axes, we have (cartesian)

$$\sigma_{ij} = -p\delta_{ij} - \frac{2}{3}\mu\delta_{ij}\theta + 2\mu D_{ij}$$

where $D_{KK} = \operatorname{div}\vec{q} = \theta$, $D_{ij} = \frac{1}{2}\left(\frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial x_i}\right)$.

Let us take $q_1 = u$, $q_2 = v$, $q_3 = w$
$x_1 = x$, $x_2 = y$, $x_3 = z$.

$$\sigma_{11} = \sigma_{xx} = -p - \frac{2}{3}\mu\theta + 2\mu\frac{\partial u}{\partial x}$$

$$\sigma_{22} = \sigma_{yy} = -p - \frac{2}{3}\mu\theta + 2\mu\frac{\partial v}{\partial y}$$

$$\sigma_{33} = \sigma_{zz} = -p - \frac{2}{3}\mu\theta + 2\mu\frac{\partial w}{\partial z}.$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\sigma_{yz} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$$\sigma_{xz} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right).$$

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