Ex: The set $\mathbb{Z}_n = \{0, 1, \ldots, n-1\}$ for $n \geq 1$ is a group under addition modulo n. Here the operation addition modulo n is ~~defined~~ denoted by $\oplus_n$ and is defined by

$a \oplus_n b = c$, where c is the least non negative integer obtained as remainder when $a+b$ is ~~defi~~ devided by 5. For example if we take $n = 3$, then

$2 \oplus_3 2 = 1$, $0 \oplus_3 5 = 2$ etc.

Ex: Let $G = \{\pm 1, \pm i, \pm j, \pm k\}$. Define product on G by usual multiplication together with,

$$i^2 = j^2 = k^2 = -1, \quad ij = -ji = k$$
$$jk = -kj = i$$
$$ki = -ik = j$$

Then G form a group and this group is called ~~as~~ Quaternian Group.

Ex: Let $G = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}, \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} \right\}$

Then G is a group under matrix multiplication.

Ex: Let $G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R}, \ ad - bc \neq 0 \right\}$

Then G is a group under matrix multiplication defined as

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1a_2 + b_1c_2 & a_1b_2 + b_1d_2 \\ c_1a_2 + d_1c_2 & c_1b_2 + d_1d_2 \end{bmatrix}$$

Here $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity element.

But the set $G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} ; \ a, b, c, d \in \mathbb{R} \right\}$ is not a group under matrix multiplication. Because if $ad - bc = 0$ that is $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0$, then $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has no inverse matrix.

Ex: Let $U(n)$ be the set of all positive integers less than n and mutually prime (or relatively prime) to n. Then $U(n)$ is a group under multiplication modulo n.

Ex: The set $\{0, 1, 2, 3\}$ is not a group under multiplication modulo 4. Since 0 and 2 do not have inverses.

Ex: The set $\mathbb{R}^n = \{(a_1, a_2, \dots, a_n) : a_1, a_2, \dots, a_n \in \mathbb{R}\}$ is a group under the componentwise addition defined as

$$(a_1, a_2, \dots, a_n) + (b_1, b_2, \dots, b_n) = (a_1 + b_1, a_2 + b_2, \dots, a_n + b_n)$$