Problem: Show that there is no simple graph corresponds ~~to~~ the following degree sequence:

(i) 0, 2, 2, 3, 4; (ii) 1, 1, 2, 3; (iv) 2, 2, 3, 4, 5, 5

(v) 2, 2, 4, 6.

Solution: Since in (i), (ii) and (iii), there are odd number of odd degree vertices in the graphs. Hence there exist no graph corresponding to this degree sequence.

(iv) The number of vertices ~~is~~ is 4.

Since the graph is simple, therefore maximum degree of a vertex in the graph ~~is~~

$= 4 - 1$

$= 3$

But, there are vertices of degree 4 and 6 in the graph. which is not possible.

$\therefore \exists$ no graph corresponds to this degree sequence.

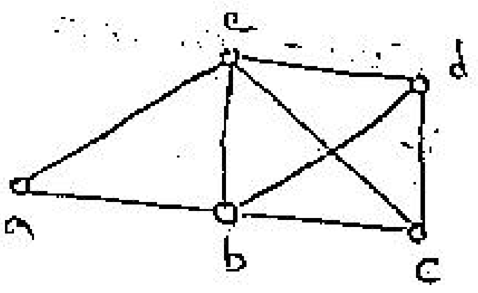
Problem: Is there a simple graph corresponding to the following degree sequences?

(i) 1, 1, 2, 3 , (ii) 2, 2, 4, 6.

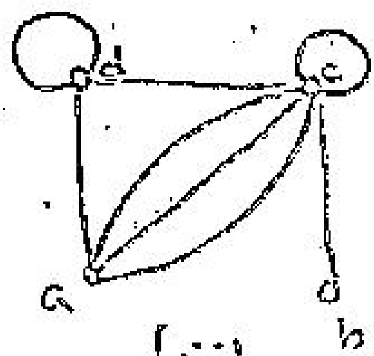
Problem: Draw the graph of the following chemical molecules

(i) Methane (CH_4) , (ii) propane (C_3H_8)

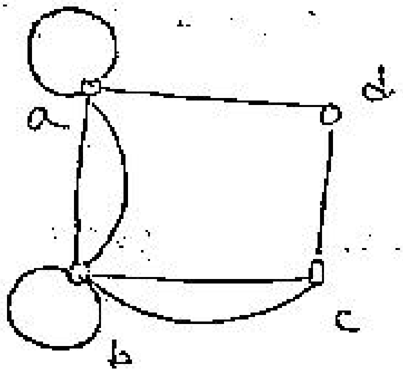
Problem: Determine the degree of each vertex of the following graphs.



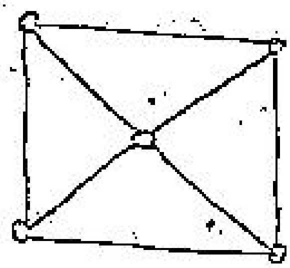
(i)



(ii)



(iii)



(iv)

problem: Draw a graph having the given properties or explain why no such graph exists.

(i) Graph with four vertices of degree 1, 1, 2 and 3.

(ii) Graph with four vertices ~~with~~ of degree 1, 1, 3, 3.

(iii) Simple graph with four vertices of degree 1, 1, 3, 3

(iv). Graph with six vertices each of degree 3.

(v) Graph with six vertices and four edges.

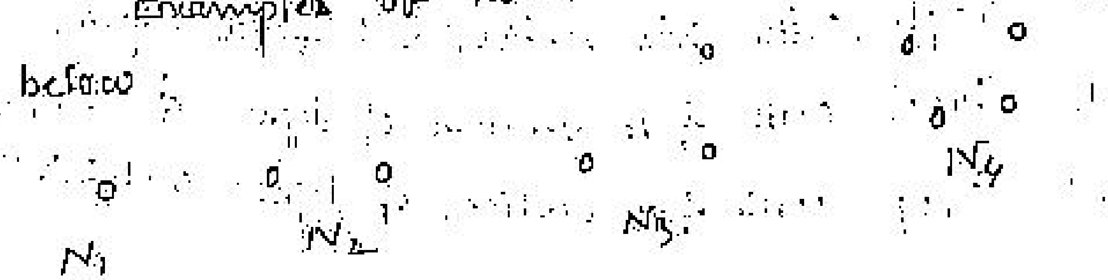
(vi) Graph with five vertices of degree 3, 3, 3, 3, 2.

(vii) Graph with five vertices of degree 0, 1, 2, 2, 3.

Types of Graphs:

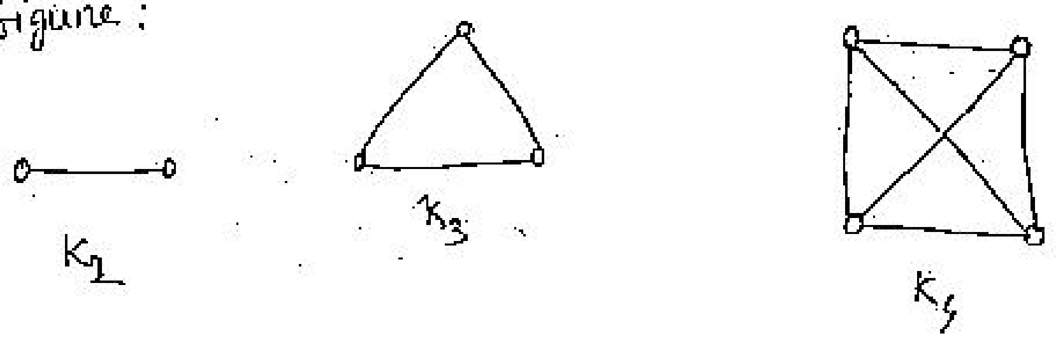
1) **Null graphs:** A graph which contains only isolated vertex is called a null graph. i.e the set of edges in a null graph is empty. A null graph on p vertices is denoted by N_p.

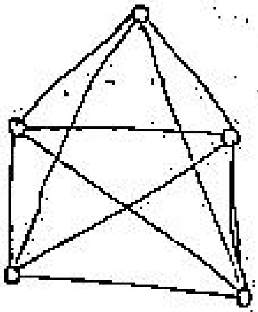
Examples of some null graphs are given below:



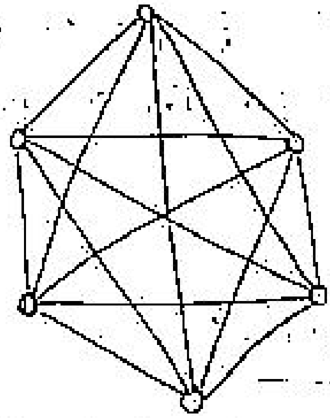
2) **Complete graph:** A graph G is said to be complete if G is simple and every ~~pair~~ vertex in G is connected with every other vertex of G. i.e A simple graph is said to be complete if it is contains exactly one edge between each pair of distinct vertices. A complete graph of n vertices is denoted by K_n.

Some complete graph are shown in the following figure:





K_5



K_6

3) Regular graph: A graph in which all vertices are of equal degree is called a regular graph. If the degree of each vertex is r, then the graph is called a regular graph of degree r or r-regular graph.

Q) Show that a regular graph of n vertices and degree r has $\frac{1}{2}rn$ edges.

Soln: Let there are q- number of edges.

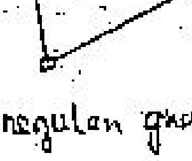
Since there are n vertices each of degree r

$\therefore$ Sum of degrees of all vertices $= 2q$

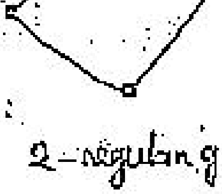
$\Rightarrow \quad n \times r = 2q$

$\Rightarrow \quad q = \frac{1}{2} rn$

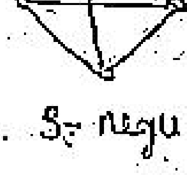
Examples of regular graph



2-regular graph



2-regular graph



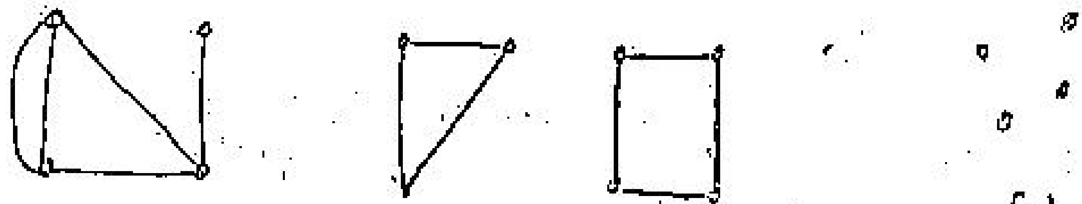
3-regular graph

Note: Every null graph is regular of degree zero and the complete graph K_n is regular of degree $n-1$.

4) **Connected graph:**

If there exist at least one ~~path~~ path between any two vertices of a graph G, then it is called a connected graph. If the graph is not connected then it is called a disconnected graph. Each of the connected part of the disconnected graph is called components.

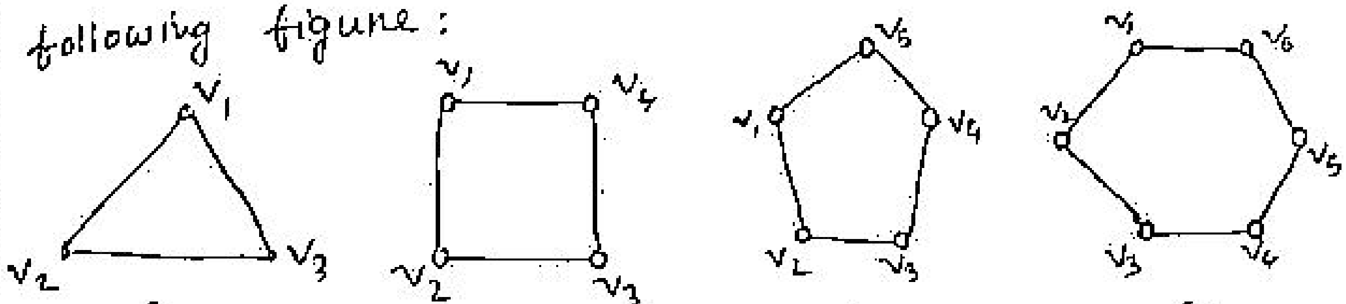
In the following figure (a) is a connected graph and (b), (c) are disconnected grap.



(a) (b) (c)

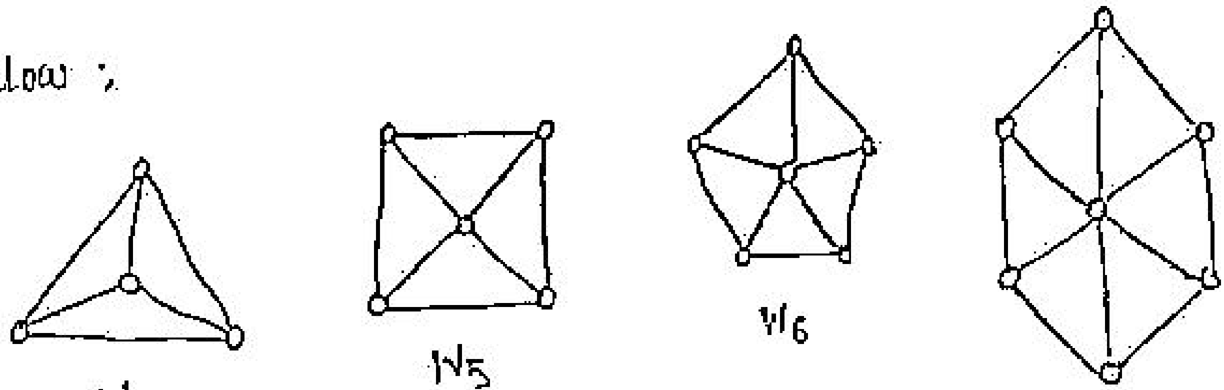
5) **Cycle graph:** A connected graph whose edges form a cycle of length n is called a cycle graph of order n. Here length n means graph consisting of n edges. Cycle graph of length n is denoted by C_n. That is the cycle C_n, $n \geq 3$, consists of n vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}$, $\{v_n, v_1\}$.

The cycle C_3, C_4, C_5, and C_6 are shown in the following figure:



C_3 C_4 C_5 C_6

6) **Wheels:** A wheel of order n is a graph obtained by joining a single new vertex to each vertex of cycle graph of order $(n-1)$. Wheels of order n are denoted by W_n. i.e the wheel W_n is obtained when an additional vertex connected each of the n vertices in C_n. The wheels W_4, W_5, W_6, W_7 are displayed below:



W_4 W_5 W_6 W_7