①

Measurement of trend

The various methods for determining trend are

(a) Freehand or graphic method

(b) Semi average method

(c) Moving average method

(d) Method of least square.

Method of least square: This is the best and the most objective method of determining trend. In this method an objective type of trend equation is at first selected and then the constants involved in the equation are estimated on the basis of the data in hand. The choice of appropriate trend equation is facilitated by a graphical representation of the data. Apart from the usual arithmatic scale, semilogarithmic or doubly logarithmic scale may be used for the graphical representation of the data.

Suppose, $x_i = x_1, x_2, x_3, \ldots, x_n$ and

$y_i = y_1, y_2, y_3, \ldots, y_n$ be the n

values of the variables X and Y.

A straight line trend is represented by the equation $Y_t = a + bX_t \longrightarrow (A)$

Let the estimated line be

$$\hat{Y}_t = \hat{a} + \hat{b} X_t$$

where $\hat{a}$ and $\hat{b}$ are estimates of the two unknowns a and b respectively.

Now $(Y_t - \hat{Y}_t)$ is called the error of estimation.

ie $E_t = Y_t - \hat{Y}_t$

$$\therefore \Sigma E_t^2 = \Sigma (Y_t - \hat{Y}_t)^2$$

The principle of Least Squares is that the values of $\hat{a}$ and $\hat{b}$ should be chosen in such a way as to make the expression $\Sigma (Y_t - \hat{Y}_t)^2$ a minimum, ie the sum of the squares of the errors becomes minimum. The estimated line is called the line of best fit, because the unknowns a and b are estimated by the method of Least Squares.

Let $S = \sum (Y_t - \hat{Y}_t)^2 = \sum (Y_t - \hat{a} - \hat{b}X_t)^2 \longrightarrow (1)$

For minimising (1), the first order (or necessary) condition is that the partial derivatives of (1) w.r.t. $\hat{a}$ and $\hat{b}$ should be equal to zero. ie

$$\frac{\partial S}{\partial \hat{a}} = 0 \quad \text{and} \quad \frac{\partial S}{\partial \hat{b}} = 0$$

$$\Rightarrow \frac{\partial}{\partial \hat{a}} \sum (Y_t - \hat{a} - \hat{b}X_t)^2 = 0 \quad \text{and} \quad \frac{\partial \sum (Y_t - \hat{a} - \hat{b}X_t)^2}{\partial \hat{b}} = 0$$

$$\Rightarrow 2\sum (Y_t - \hat{a} - \hat{b}X_t)(-1) = 0 \qquad \Rightarrow 2\sum (Y_t - \hat{a} - \hat{b}X_t)(-X_t) = 0$$

$$\Rightarrow \sum (Y_t - \hat{a} - \hat{b}X_t) = 0 \qquad \Rightarrow \sum X_t Y_t - \sum \hat{a} X_t - \sum \hat{b} X_t^2 = 0$$

$$\Rightarrow \sum Y_t - \sum \hat{a} - \sum \hat{b} X_t = 0$$

$$\Rightarrow \sum Y_t = n\hat{a} + \hat{b}\sum X_t \longrightarrow (II) \qquad \Rightarrow \sum X_t Y_t = \hat{a}\sum X_t + \hat{b}\sum X_t^2 \longrightarrow (III)$$

Equations (II) and (III) are called normal equations for determining the values of $\hat{a}$ and $\hat{b}$. By substituting the values of $\hat{a}$ and $\hat{b}$ in the equation $Y = \hat{a} + \hat{b}X_t$, We get the required (trend) line of best fit or regression line of Y on X.