

$$x_5 = 2.9431$$

Comparison x_5 & $x_4 = 2.943$

$$\text{final root} = 2.943$$

Regula Falsi

$$x_4 = 2.9427$$

Put x_4 in $f(x)$

$$f(x) = x^3 - 9x + 1$$

$$f(x_4) = (2.9427)^3 - 9(2.9427) + 1$$

$$f(x_4) = -2.038 \times 10^{-3} \quad \text{-ve}$$

$$x_5 = x_0 - \frac{(x_1 - x_0) \cdot f(x_0)}{f(x_1) - f(x_0)}$$

$$x_1 = 3$$

$$f(x_1) = 1$$

$$x_0 = 2.9427$$

$$f(x_0) = -2.038 \times 10^{-3}$$

$$x_5 = 2.9427 - \frac{(3 - 2.9427) \cdot (-2.038 \times 10^{-3})}{(1 + 2.038 \times 10^{-3})}$$

$$x_2 = 2.9 - \frac{(3 - 2.9)}{(1 + 0.711)} \cdot (-0.711)$$

$$x_3 = 2.9415$$

Put x_3 in $f(x)$

$$f(x) = x^3 - 9x + 1$$

$$f(x_3) = (2.9415)^3 - 9(2.9415) + 1$$

$$f(x_3) = -0.0223 \quad \text{-ve}$$

$$x_4 = x_3 - \frac{(x_3 - x_0)}{f(x_3) - f(x_0)} \cdot f(x_3)$$

$$x_1 = 3 \quad f(x_1) = 1 \quad x_0 = 2.9415 \quad f(x_0) = -0.0223$$

$$x_4 = 2.9415 - \frac{(3 - 2.9415)}{(1 + 0.0223)} \cdot (-0.0223)$$

Regula falsi method the root lies b/w 2.23.

$$x_4 = 2.9427$$

Put x_4 in $f(x)$

$$f(x) = x^3 - 9x + 1$$

$$f(x_4) = (2.9427)^3 - 9(2.9427) + 1$$

$$f(x_4) = -2.038 \times 10^{-3} \quad \text{-ve}$$

$$x_5 = x_0 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} \cdot f(x_0)$$

$$x_1 = 3 \quad f(x_1) = 1$$

$$x_0 = 2.9427 \quad f(x_0) = -2.038 \times 10^{-3}$$

$$x_5 = 2.9427 - \frac{(3 - 2.9427)}{(1 + 2.038 \times 10^{-3})} \cdot (-2.038 \times 10^{-3})$$

Qwe ① → find a root $f(x) = x^3 - 9x + 1 = 0$ by Regula falsi method the root lies b/w 2 & 3.

solⁿ ① → $f(x) = x^3 - 9x + 1 = 0$

$x = 2 \rightarrow f(2) = 8 - 18 + 1 = -9 = -ve$

$x = 3 \rightarrow f(3) = 27 - 27 + 1 = 1 = +ve$

$x_0 = 2 \rightarrow f(x_0) = -9$

$x_1 = 3 \rightarrow f(x_1) = 1$

$$x_2 = x_0 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} \cdot f(x_0)$$

$$x_2 = 2 - \frac{(3 - 2)}{1 + 9} \cdot (-9)$$

$$x_2 = 2 + \frac{1}{10} \times 9$$

$$x_2 = 2 + \frac{9}{10}$$

$$x_2 = 2 + 0.9$$

$x_2 = 2.9$ ✓

Put $x_2 = 2.9$ in $f(x)$

$$f(x) = x^3 - 9x + 1$$

$$f(x_2) = (2.9)^3 - 9 \times 2.9 + 1$$

$f(x_2) = -0.711$ -ve

$$x_3 = x_0 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} \cdot f(x_0)$$

$x_1 = 3$

$f(x_1) = 1$

$x_0 = 2.9$

$f(x_0) = -0.711$

Second Approx.: Put $x_0 = 0.31466$ & $x_1 = 1$

$$x_3 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_0)$$
$$= 0.44672$$

$$f(x_3) = -0.20354$$

A root lies betⁿ 0.44672 & 1 .

~~x_4~~

Repeat the process, the successive approximations are

$$x_4 = 0.49401$$

$$x_6 = 0.51520$$

$$x_8 = 0.51748$$

$$x_5 = 0.50995$$

$$x_7 = 0.51692$$

$$x_9 = 0.51767 \approx 0.5177$$

$$x_{10} = 0.51774 \approx 0.5177$$

Problem 1. Find a root of the equation $x e^x = \cos x$ using the regula-falsi method correct to four decimal places.

Solⁿ Let $f(x) = x e^x - \cos x$

$f(0) = -1, f(1) = 1 \cdot e^1 - \cos(1 \times 57.3) = 2.179 = \text{+ve}$

Root lies bet.ⁿ 0 and 1.

First approx.: ~~let~~ Put $x_0 = 0, x_1 = 1$ } The root lies bet.ⁿ

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_0)$$

$$= 0 - \frac{1 - 0}{2.179 + 1} \cdot (-1) = 0.31466$$

$$f(x_2) = (0.31466) e^{0.31466} - \cos(0.31466 \times 57.3) = -0.51988$$

Problem 1. Find a root of the equation $x e^x = \cos x$ using the regula-falsi method correct to four decimal places.

$$\frac{180}{\pi} = 57.3$$

Solⁿ Let $f(x) = x e^x - \cos x$
 $f(0) = -1$, $f(1) = 1 \cdot e^1 - \cos(1 \times 57.3) = 2.179 = \text{+ve}$

Root lies betⁿ 0 and 1.

First approx.: ~~let~~ Put $x_0 = 0$, $x_1 = 1$

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_0)$$

$$= 0 - \frac{1 - 0}{2.179 + 1} \cdot (-1) = 0.31466$$

$$f(x_2) = (0.31466) e^{0.31466} - \cos(0.31466 \times 57.3) = -0.51988$$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \underline{\hspace{2cm}}$$

$$f(x_2) = \underline{\text{+ve}}$$

$\therefore f(2) = -ve$ and $f(x_2) = +ve$,
the root lies b/w 2 and x_2

$$x_3 = \frac{(2) f(x_2) - x_2 f(2)}{f(x_2) - f(2)} = \underline{\hspace{2cm}}$$

and repeat till desired accuracy is achieved.

REGULA - FALSI METHOD (or False-Position Method)

Let $f(x)=0$ be the given eq.ⁿ

$$\begin{aligned}\text{Suppose, } f(0) &= -ve \\ f(1) &= -ve \\ f(2) &= -ve \\ f(3) &= +ve\end{aligned}$$

$\therefore f(2) = -ve$ and $f(3) = +ve$,
the root lies between 2 and 3.

$$\begin{aligned}\text{Let } x_0 &= 2 \text{ and } x_1 = 3 \\ \therefore f(x_0) &= -ve \text{ and } f(x_1) = +ve.\end{aligned}$$