Network Theorem
Lecture 10

Manoj Kr. Das

Associate Professor

Physics Department

J N College, Boko

Introduction:

Electrical circuit theorem always beneficial to help find voltage and current in multi loop circuits. These therems use fundamental rules or formula and basic equation of mathematics to analyze the basic components of electric or electrical parameters such as voltages, currents and so on. These fundamental theorems include basic theorem as like Superposition Theorem, Norton’s Theorem, Tellengen’s Theorem, Maximum Power Transfer Theorem, Thevinin’s Theorem. Another group of network theorems that are mostly used in the circuit analysis process includes the Compensation, Substitution Theorem, Reciprocity Theorem, Millman’s Theorem and Miller’s Theorem.

Matrix Method and Circuit Analysis:

Let three current equation are

$$Z_{11}I_1 \pm Z_{12}I_2 \pm Z_{13}I_3 = V_1 \rightarrow (i)$$

$$\pm Z_{21}I_1 \pm Z_{22}I_2 \pm Z_{23}I_3 = V_2 \rightarrow (ii)$$

$$\pm Z_{31}I_1 \pm Z_{32}I_2 + Z_{33}I_3 = V_3 \rightarrow (iii)$$

These can be written as in matrix form

$$\begin{bmatrix} Z_{11} & \pm Z_{12} & \pm Z_{13} \\ \pm Z_{21} & Z_{22} & \pm Z_{23} \\ \pm Z_{31} & \pm Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \rightarrow (iv)$$

$$[Z]\,[I] = [V] \rightarrow (v)$$

Here $[V]$ is voltage matrix, $[Z]$ is impedance matrix and $[I]$ is current matrix. This is known matrix form of Ohm's Law. Obviously I_1 , I_2, I_3 can be expressed as

$$I_1 = \frac{\begin{vmatrix} V_1 & \pm Z_{12} & \pm Z_{13} \\ V_2 & Z_{22} & \pm Z_{23} \\ V_3 & Z_{32} & Z_{33} \end{vmatrix}}{\Delta Z} \rightarrow (vi)$$

$$I_2 = \frac{\begin{vmatrix} Z_{11} & V_1 & \pm Z_{13} \\ \pm Z_{21} & V_2 & \pm Z_{23} \\ \pm Z_{31} & V_3 & Z_{33} \end{vmatrix}}{\Delta Z} \rightarrow (vii)$$

$$I_3 = \frac{\begin{vmatrix} Z_{11} & \pm Z_{12} & V_1 \\ \pm Z_{21} & Z_{22} & V_2 \\ \pm Z_{31} & \pm Z_{32} & V_3 \end{vmatrix}}{\Delta Z} \rightarrow (viii)$$

If numerator determinant of each is expanded about the elements of the column containing voltages, one can get the following st of equations for mesh current.

$$I_1 = V_1\left(\frac{\Delta_{11}}{\Delta Z}\right) + V_2\left(\frac{\Delta_{21}}{\Delta Z}\right) + V_3\left(\frac{\Delta_{31}}{\Delta Z}\right) \rightarrow (ix)$$

$$I_2 = V_1\left(\frac{\Delta_{12}}{\Delta Z}\right) + V_2\left(\frac{\Delta_{22}}{\Delta Z}\right) + V_3\left(\frac{\Delta_{32}}{\Delta Z}\right) \rightarrow (x)$$

$$I_3 = V_1\left(\frac{\Delta_{13}}{\Delta Z}\right) + V_2\left(\frac{\Delta_{23}}{\Delta Z}\right) + V_3\left(\frac{\Delta_{33}}{\Delta Z}\right) \rightarrow (xi)$$

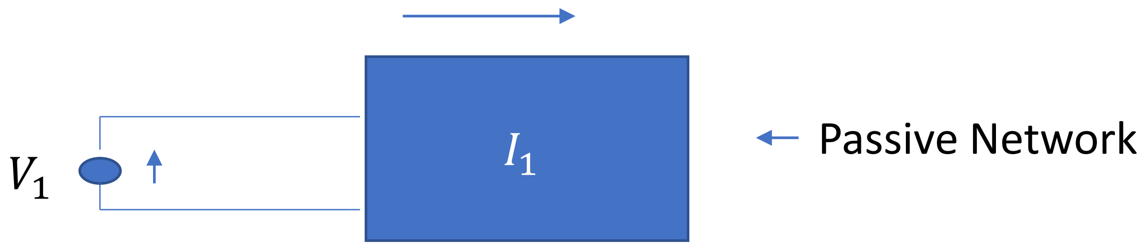

Fig 1

Let us consider a source free network with two external connections. A voltage source V_1 be applied to external connection. Here

$$I_1 = V_1\left(\frac{\Delta_{11}}{\Delta Z}\right) + 0.\left(\frac{\Delta_{21}}{\Delta Z}\right) + 0.\left(\frac{\Delta_{31}}{\Delta Z}\right) + \cdots \rightarrow (xii)$$

$$I_1\ [= V_1\left(\frac{\Delta_{11}}{\Delta Z}\right)$$

$$\frac{V_1}{I_1} = \frac{\Delta Z}{\Delta_{11}} \rightarrow (xiii)$$

The relation $\frac{V_1}{I_1}$ is known as Input or Drawing Point Impedance. This is the impedance to specified terminals when all inter sources are shorted, but internal impedance are retained.

Fig 2

Here I_2 is the recurring current.

The Transfer Impedance is defined as ratio of driving voltage in one mesh to resulting current at another when other sources are made zero.

$$I_2 = 0 + \cdots + V_2\left(\frac{\Delta_{12}}{\Delta Z}\right) + \cdots + 0\left(\frac{\Delta_{12}}{\Delta Z}\right) \rightarrow (xiv)$$

$$I_2 = V_2\left(\frac{\Delta_{12}}{\Delta Z}\right)$$

$$\frac{V_2}{I_2} = \frac{\Delta Z}{\Delta_{12}} \rightarrow (xv)$$

This is known as Transfer Impedance. Δ_{12} means source is mesh 1 and resulting current in mesh 2.

Superposition Theorem:

Statement: The current through or voltage across an element in a linear bilateral network is equal to the algebraic sum of currents or voltages produced independently by each source.

The superposition theorem is a way to determine the currents and voltages present in the circuit that has multiple sources (considering one source at a time). The superposition theorem states that in a linear network having a number of voltage or current sources and resistances, the current through any branch of the network is the algebraic sum of currents due to each of sources when acting independently. Superposition theorem is used only in case of linear network. This theorem is used in both AC or DC circuit.

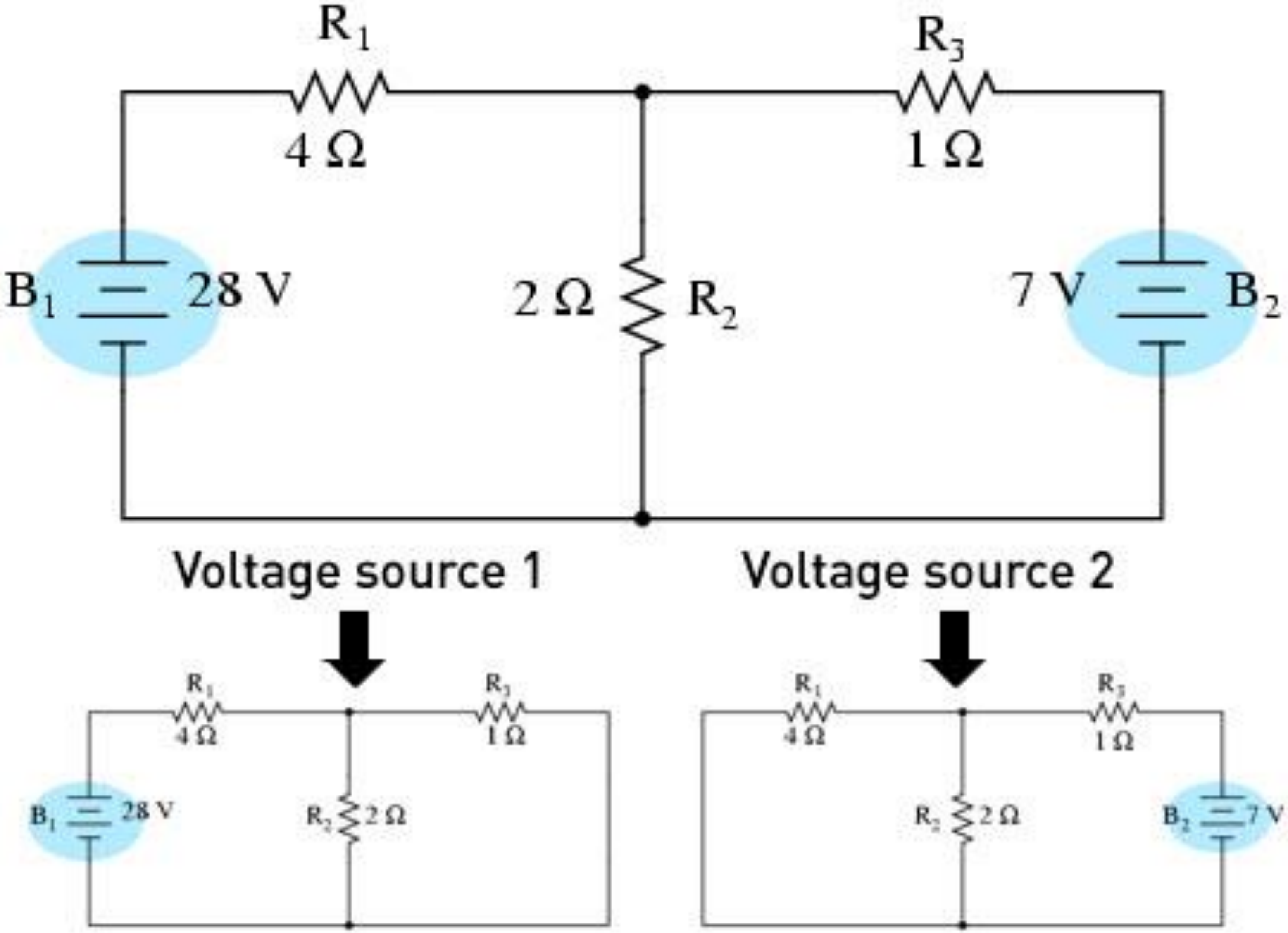

Fig 3

It helps to construct Thevenin and Norton’s equivalent circuit. As shown in Fig 1 the circuit with two voltage sources is divided into two individual circuits according to the theorem’s statement.

The individual circuits here make the whole circuit make look simpler in easier way and combining the two circuits again after individual simplification, one can easily find parameters like voltage drop at each resistance, node voltage, currents e.t.c.

When one applying this theorem, it is possible to consider the effects of two sources at the same time and reduce the number of networks that have to be analysed but in general.

$$Number\ of\ network\ to\ be\ analyze = Number\ of\ independent\ sources$$

To consider the effect of each source independently requires that sources to be removed and replaced without effecting the final result.

To remove a voltage source when applying this theorem, the difference in potential between the terminals of the voltage source must be set to zero (short circuit), removing a current source requires that its terminals to be open (open circuit).

Any internal resistance or conductance associated with the displaced sources is not eliminated but must be considered.