VECTOR ANALYSIS - II

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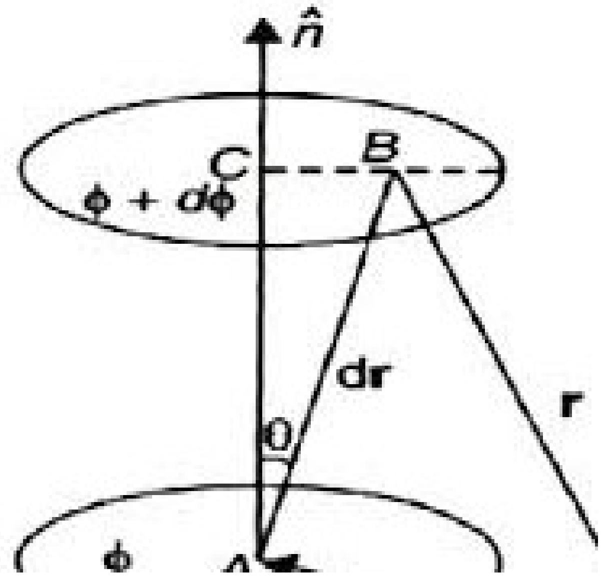
GRADIENT OF A SCALAR FIELD

The **gradient of a scalar** field is a vector field and whose magnitude is the rate of change and which points in the direction of the greatest rate of increase of the **scalar** field.

Gradient is a vector that represents both the magnitude and the direction of the maximum space rate of increase of a **scalar**.



GRADIENT OF A SCALAR FIELD

- The gradient of a scalar function $f(x_1, x_2,$
denoted by ∇f or where ∇ (the nabla symbol)
vector differential operator, del. The notation

directional derivative of f along **v**. That is,

$$(\nabla f(x)) \cdot \mathbf{v} = D_{\mathbf{v}} f(x)$$

3-dimensional cartesian c
noted by:

Q. If $\phi = x^2yz^3 + xy^2z^2$,determine $grad\phi$ at point P=

solution

$$\phi = x^2yz^3 + xy^2z^2$$

$$\frac{\partial\phi}{\partial x} = 2xyz^3 + y^2z^2, \frac{\partial\phi}{\partial y} = x^2z^3 + 2xyz^2, \frac{\partial\phi}{\partial z} = 3x^2yz$$

Therefore,

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$= \left(2xyz^3 + y^2z^2\right)\hat{i} + \left(x^2z^3 + 2xyz^2\right)\hat{j} + \left(3x^2yz^2 + 2\right.$$

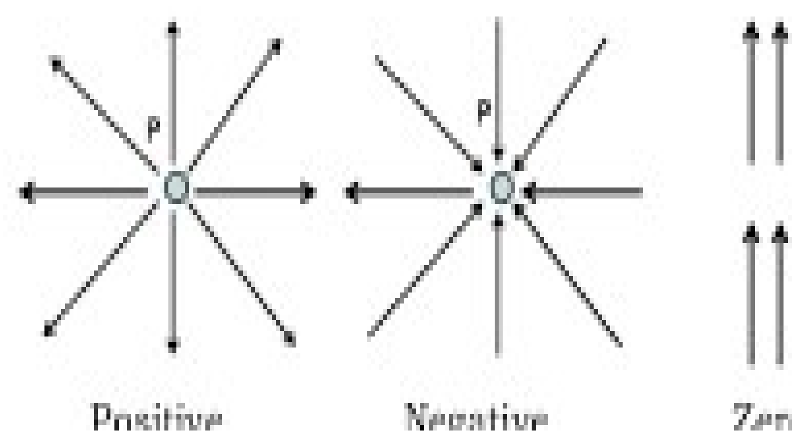
at

Divergence of a Vector Field

The **divergence of a vector field** simply measures how much the flow is expanding at a given point. It does not indicate in which direction the expansion is occurring. The **divergence** is a scalar.

Divergence of a Vector Field is define as the Net amount of Flux diverging or converging per unit volume at a point

Illustration of the divergence of a vect field at point P:



DIVERGENCE

- In terms of the gradient operator

$$\nabla = \left(\frac{\partial}{\partial x}\right)\mathbf{i} + \left(\frac{\partial}{\partial y}\right)\mathbf{j} + \left(\frac{\partial}{\partial z}\right)\mathbf{k}$$

t

- The divergence of F can be written symbolica product of and F:

If

$$\text{div}\, \boldsymbol{F} = 0$$

Then *F* is called Solenoidal Vector.

Divergence of a vector

If $A = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$,the divergence of A is defined as

$$divA = \nabla . A$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) \cdot \left(a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \right)$$

$$\Rightarrow \frac{\partial a_x}{} + \frac{\partial a_y}{} + \frac{\partial a_z}{}.$$

Q. If $A = x^2 y\hat{i} - xyz\hat{j} + yz^2\hat{k}$, determine $divA$ at poir

<u>solution</u>

$$divA = \nabla . A$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) \cdot \left(a_x \hat{i} + a_y \hat{j} + a \right.$$

$$\Rightarrow \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}.$$

$$divA = 2xy - xz + 2yz$$

$$at(1, 2, 3)$$

$$divA = 2(1)(2) - (1)(3) + 2(2)(3)$$

Curl of a vector

If $A = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$,the curl of A is defined by

$$curl A = \nabla \times A$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) \times \left(a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \right.$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial & \partial & \partial \end{vmatrix}$$

Curl A = 0, A is called irrotational Vector

Q. If $A=(y^4-x^2z^2)\hat{i}+(x^2+y^2)\hat{j}-x^2yz\hat{k}$,determine c

at (1,3,-2).

<u>solution</u>

$$curlA=\nabla\times A=\begin{vmatrix} \hat{i} & \hat{j} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ y^4-x^2z^2 & x^2+y^2 & - \end{vmatrix}$$

$$=-x^2zi-(-2xyz+2x^2z)\hat{j}+(2x-4y^3)$$

$$At(1,3,-2),$$

THANK YOU

ALL THE BEST