Motion at low Reynold number:

Navier Stokes equation in non-dimensional form is

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\vec{\nabla})\vec{q} = -\Delta p + \frac{1}{R}\nabla^2\vec{q} \longrightarrow (i)$$

There are some Liquid flows when $\vec{q}$ and its derivatives are small in magnitude i.e. q^2 or the products like uv, vw, ... etc are negligible. In this case, the Reynold number $\frac{UL}{\nu}$ is small for the typical velocity U is small. Hence the viscous term $\frac{1}{R}\nabla^2\vec{q}$ is more important than the inertia term $(\vec{q}\cdot\vec{\nabla})\vec{q}$. So, for slow motion Navier stokes equation (1) is approximate to

$$\frac{\partial \vec{q}}{\partial t} = -\nabla p + \frac{1}{R}\nabla^2\vec{q} \longrightarrow (ii)$$

Taking divergence of it, we have,

$$\nabla\cdot\frac{\partial \vec{q}}{\partial t} = -\nabla^2 p + \frac{1}{R}\nabla\cdot\left[\nabla^2\vec{q}\right]$$

$$\text{i.e. } \frac{\partial}{\partial t}(\nabla\cdot\vec{q}) = -\nabla^2 p + \frac{1}{R}\nabla^2(\nabla\cdot\vec{q})$$

For liquid $\text{div}\,\vec{q} = 0$, this equation reduces to

$$\nabla^2 p = 0$$

$\therefore$ p satisfies Laplace's equation.

Assuming harmonic solution of pressure P, solution for u, v, w (i.e. $\vec{q}$) with appropriate boundary condition are satisfied.

Stokes first assumed that for slow motion of solid, the non-Linear term $(\vec{q}\cdot\nabla)\vec{q}$ is negligible in comparison to $\frac{1}{R}\nabla^2\vec{q}$. This approximation is known as stoke's approximation or stoke's Linearisation process. Stoke's Linearisation process has limitations. This cannot be applied to discuss the motion of fluid past a circular cylinder. One undetermined arbitrary constant makes the solution infinite. This is known as stoke's paradox.

Again, whitehead tried to proved this result of stoke's solution past a sphere by the method of iteration. He took

$$\psi = \psi_0 + \psi_1 + \psi_2 + \cdots$$

where $D^4\psi_0 = 0$.

He shows that ψ gives infinite values of velocity at infinity i.e. stoke's solution cannot be improved by the method of iteration.
